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Depreciated Depreciation Methods? Alternatives to Sraffa's Take on Fixed Capital

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This article discusses the treatment of fixed capital in Classical theory of price. Sraffa uses non-linear depreciation of "physical" capital that equalizes all annual profit rates individually, but violates the proportionality of monetary machine value reduction and physical use-up on an annual basis. One alternative is to apply simple linear depreciation that has equal annual fixed capital costs. The key for consistency is that the internal rate of return on fixed capital investments throughout the fixed asset lifetime must be equated with the normal profit rate. A second alternative is to use "monetary" capital, where the "correct" amortization charges depend on the ability of the accumulated depreciation fund to earn interest. Among these valid alternative methods are the original proposals of Marx and Torrens, which were dismissed falsely and prematurely by Neo-Ricardian economists. These alternatives are shown here to imply fundamentally different prices of production. For all methods, the formulas for deriving amortization charges and fixed capital prices of all vintages are derived. The article also illustrates how the system of Sraffian price equations can be modified to incorporate these methods.

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I. Introduction

The analysis of fixed capital and its representation in price theory is one of the major occupations for Neo-Ricardians over the last fifty years (e.g. Pasinetti, 1980; Schefold, 1989; Sraffa, 1960). The determination of depreciation of fixed capital plays a role in historical controversies (e.g. between Steedman (1980) and Shaikh (1980)) and in more current discussions (see Duménil and Lévy (2000) and Freeman (2004) or Moseley (2009, 2011) and Gehrke (2011)) and in recent analyses (see e.g. Lager, 2006; Vitaletti, 2008; Shaikh, 2015).

A source of possible confusion is the fact that three different problems are involved with the inclusion of fixed capital in Classical pricing systems. While these problems are all unrelated to each other, their separation has not been made clear in the debates between authors like Moseley (2009, 2011) and Gehrke (2011), usually in the sense that joint production is associated with Sraffa's common "profitability consistent method" and "transfer of value" with linear depreciation.[1]

One goal of this article is to analyze these existing three problems individually, to show that they are independent and to attempt a clarification (section II). The first complication is the "technical problem of fixed capital", that is how to write depreciation and fixed capital in a formal mathematical model or equation system (joint production vs. "transfer of value"). It is logically independent of the two "economic problems of fixed capital", but it has always been present and incorrectly aligned with them. The first economic problem is a fundamental issue that has not been formulated clearly in the past. It is a fact that it is mathematically impossible to simultaneously satisfy three core principles of prices of production: 1. the law of one price; 2. the equalization of annual profit rates;

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[1] However, both authors do acknowledge, for example, Ricardo's support of the transfer of value approach (it is generally known that he also applied the profitability consistent method). Nevertheless, an explicit proper clarification where both problems are perfectly separated, is missing.

3. the proportionality between price and cost of production in every production period. I argue that strong and weak versions of the latter two principles can be formulated and that the weak ones must not be violated by any "correct" solution. The second economic problem is the identification of the capital quantity that is used in the computation of whatever profit rate is to be equalized. It could be either just the value of fixed assets that remains invested in the industry after depreciation ("*physical*" capital) or the sum of this and the accumulated depreciation fund to which the annual depreciation charges add up to ("*monetary*" capital). If it is the latter, then one needs to clarify the ability of the accumulated depreciation fund to receive interest. The two economic problems of fixed capital are not only independent of the technical one. They are also unrelated to each other in the sense that every possible consistent answer to the first problem can be combined with each of the different capital concepts that answer the second problem.

The common Sraffian solution is to rely on the physical capital concept and on strong "*annual profit rate consistency*". Using the standard annuity loan formula, it sets all individual annual profit rates of all vintages of fixed capital equal to the normal rate of return. However, it destroys the proportionality between price and cost of production in every individual year and only allows for a weak "*lifetime cost consistency*" where the sum of all depreciation charges throughout the fixed asset lifetime is equal to the initial price of fixed capital.

The alternative proposal of Torrens, Marx and other Classical authors is "*annually cost consistent*", where all annual depreciation charges are set equal to the rate of physical deterioration of the fixed asset. With a linear use-up of physical capital, this yields constant fixed capital unit costs. However, it appears to be flawed in its common representation and is swiftly dismissed by almost all authors, as either equilibrium prices change with fixed capital age (Gehrke, 2011), or the individual annual profit rates of every vintage of a fixed asset are unequal to one another and unrelated to the normal rate of profit (see e.g. Lager, 2006;

Kurz and Salvadori, 1997).[2] One contribution of the article is to show that this "simple" linear depreciation method of these Classical economists is correct under the assumptions of monetary capital and a simple credit system, where the depreciation fund earns no interest.

When the annual fixed cost is constant, it is only possible to equalize the single annual profit rates on fixed assets if the interest rate on the depreciation fund is zero. An important innovation of this article is to show that this problem can always be solved for any interest rate by setting the *internal rate of return* on the fixed asset's annual amortization charges (that accrue throughout the fixed asset lifetime) equal to the normal rate of return. This "*lifetime profit rate consistent*" solution is the mirror image of Sraffa's annual profit rate consistent method: both use annuity formulas to find the "correct" amortization charge. One has annual profit rate consistency while the other has annual cost consistency. The former is only lifetime cost consistent while the latter is lifetime profitability consistent. I argue that lifetime profit rate consistency should be accepted as a criterion for any "correct" method in addition to the known lifetime cost consistency requirement.

The article is the first to derive all exact formulas for computing annual amortization charges and fixed capital prices for all vintages for every possible combination of cost/profit rate consistency on the one hand and physical/monetary capital on the other hand (section III.B). Sraffa himself saw room for a multitude of correct depreciation schemes as long as the law of one price and the intratemporal cost consistency are not violated.[3] The fact that he revised his treatment of depreciation several times throughout the course of his work shows that he had doubts himself about how to solve the problem properly. Solutions are derived here also for all versions of the depreciation fund (different abilities to

[2]Schefold (1989, p. 179) who analyzes and defends the method as a potentially valid approximation of the common Neo-Ricardian approach is an exception.

[3]"There are innumerable schemes of dividing the fixed annuity of Fixed Cap. between replacement of capital and profit. All these schemes are equally correct if they satisfy the requirements: 1) That the total paid in each successive year is constant. 2) That the total paid for replacement of capital at any time is equal (without the addition of any interest) to the original cost of the asset." Sraffa quoted by Kurz and Salvadori (2005, p. 513).

earn interest). It is shown how the Sraffian price equations need to be adjusted to incorporate the proposed alternative methods of treating fixed capital.

Two new insights are derived that contradict ideas formulated by Sraffa (Kurz and Salvadori, 2005, p. 513). First, the "correct" specific financial mathematical formulas for any of the two capital concepts (also the physical one) and for any interest rate on the depreciation fund are mathematically unique when "correct" implies lifetime cost consistency and (simultaneously) annual profitability consistency or annual cost consistency and (simultaneously) lifetime profitability consistency. It is not generally true that *any* arbitrary division of the annual amortization charge between profit and depreciation is "correct". Second, switching from strict profitability consistency to strict cost consistency affects the annual amortization charge and the prices of production when one assumes monetary capital that has the ability to earn interest. Switching from a physical to a monetary capital concept also implies different charges and prices. These insights prove that the issue at hand is not just a negligible question of taste in accounting matters, but one that is relevant for the determination of the actual equilibrium in Classical economics.

There are several issues that this article cannot address, such as the more realistic effect of changing or variable efficiency of capital and maintenance costs throughout time and the resulting truncation problem; the method of depreciation by radioactive decay or other depreciation phenomena like "moral depreciation" that results from assets becoming obsolete due to technical improvements of new machines.[4] These important matters must be subject to future research. To rule out any potential misunderstandings, it must also be highlighted in the strongest possible way, that the purpose of this article is not to dismiss Sraffa's solution as inferior. The sole point is that it is not the only consistent method and that other, previously broadly dismissed approaches can also be perfectly valid.

[4]The assumption of constant efficiency and spontaneous "implosion" of the asset is a major simplification and not very realistic for many assets. However, it is made here nevertheless, since it is standard in the literature which this article speaks to.

Whether monetary or physical capital is preferred or whether strong annual cost consistency should be chosen over strong annual profit rate consistency should be subject to a future discourse. Its outcome might even render some or all but one solution as generally inferior. With respect to the different problems of fixed capital and to the solutions formalized here, it must also be mentioned that these issues are already being discussed in the literature. They do not represent new positions. Instead, an attempt is made here to clarify the points, formulate views in an analytical and quantifiable way. This allows to generate new insights and derive different mathematical solutions for prices of production that correspond to the different answers to these open questions about the treatment of fixed capital.

II. Three Problems of Fixed Capital

In the following, several simplifying assumptions are made: a period of production is equal to one year; labor costs accrue at the end of the year; there is no proper joint production, meaning that apart from the ability to use different vintages of the same machine, there is one and only one production process to produce a commodity (efficiency and utilization are constant for every production period during the lifetime of assets); fixed capital is operated without maintenance, repair, or replacement of parts; depending on the example, production requires the use of either no or only one fixed capital asset; "amortization" is defined as the sum of depreciation and profit charges on fixed capital; there is free disposal (machines can be eliminated at zero cost) and zero value[5] of scrap. In equations, bold lower case letters represent vectors, bold upper case letters matrixes and regular (lower or upper case) letters are scalars.

[5] Unless explicitly stated otherwise the term "value" is not referring to "labor values" in this article but to the monetary value that is based on prices of production.

A. The Technical Problem of Fixed Capital

The "technical problem" of fixed capital is of a technical nature in the sense that it models for one fixed capital asset the same actual productive use, ownership allocation and the same set of market transaction processes. Identical output prices, depreciation charges, profit charges and prices for allvintages of the capital asset are derived in different ways. Two solutions have been suggested in the literature.

The first one is the joint production approach proposed by Torrens (1818, pp. 336-337) and found in Torrens (1821, pp. 28-29), Malthus (1825, p. 305), one passage of the third edition of the Principles (Ricardo, 1951, pp. 33) and in Marx (1990, p. 321).[6] Sraffa's formalization in a pricing equation for an individual commodity that relies on a single fixed capital input is

$$wl + (1 + r^*)(\boldsymbol{p_a a} + p_{k_t} k_t) = p_b b + p_{k_{t+1}} k_{t+1} \tag{1}$$

where w and r^* are the normal wage and profit rates, l the labor input requirement for the commodity, $\boldsymbol{a}$ the vector of material input requirements and $\boldsymbol{p_a}$ the corresponding input price vector. b and p_b are output and its price respectively. k_t and p_{k_t} represent the fixed capital of a certain vintage and its respective price. There is a total of T such processes if the fixed asset has a lifetime of T years. Each represents the production with one of the possible vintages of the fixed asset. For simplicity, assume that there is only labor and one fixed capital input unit in every process. Rewriting $p_{k_t} k_t = K_t$ simplifies equation 1 to

$$wl + (1 + r^*)K_t = p_b b + K_{t+1} \tag{2}$$

As in any scheme, depreciation charges here also equal the difference in prices for

[6] Moseley (2011) disputes that Marx and Ricardo adopted the joint production method while Sraffa (1960), Gehrke (2011) and others believe that they did.

fixed capital of consecutive vintages,

$$D_t = K_t - K_{t+1} \tag{3}$$

The second solution is usually called the "transfer of value" approach.[7] It is used in all editions of the Principles (Ricardo, 1951, pp. 35, 60, 432) and in Marx (1992, e.g. pp. 239, 243). The fixed capital asset value as a whole is not included in pricing equations. It only appears as the base from which a profit charge is computed. The fixed capital principal does not enter any "valorization" process – not even an imagined or a hypothetical one for pure accounting matters. Only the fraction of their quantities that is used-up in the current production period appears in the form of depreciation and only on the left side of the price equation. The formal representation in corresponding pricing equations is less common, but more straight forward. The purely technical nature of the choice between the two methods is clear when shown mathematically: the exact solution to the economic problems of fixed capital in equation 2 can be implemented also with the transfer of value approach. For this, one can just rearrange equation 2 by subtracting K_{t+1} from both sides and using equation 3 to substitute D_t for $K_t - K_{t+1}$. This yields

$$wl + r^* K_t + D_t = p_b b \tag{4}$$

Obtaining identical results in equations 2 and 4 only requires that the prices of different vintages of fixed capital be determined in the same way. If they are derived from the standard annuity formula, both illustrations yield Sraffa's solution. Equivalently, both technical schemes can represent simple linear depreciation if $K_t = K_1 - t\frac{K_1}{T}$ and if r^* is replaced by r_t.[8] In fact, it turns out that both so-

[7] The term is unfortunate as it suggests a treatment in terms of (labor) values as opposed to prices of production which is of course not the difference to the joint production method.

[8] For a consistent solution where the internal rate of return is normalized, one needs to have a profit charge on the remaining fixed capital stock invested that is different in every year, r_t, and not equal to r^*.

lutions to the technical problem have been combined in the literature with all possible answers to the first economic problem of depreciation[9]: 1. annual profit rate consistency and joint production (Ricardo, Sraffa); 2. annual profit rate consistency and transfer of value (Ricardo, Bortkiewicz); 3. annual cost consistency and joint production (Torrens, Marx); 4. annual cost consistency and transfer of value (Marx). The equivalence of both technical representations was already stated verbally by Marx (1990, pp. 320-321) and proved formally by Perri (1990).

This shows an essential point, which is of paramount importance for dealing with fixed capital in Classical price theory: the representation of fixed capital as a joint product is a mere analytical tool to technically process the complexities in finding natural prices. It was never supposed to be an actual description of a real production process of vintage fixed assets. None of the major Neo-Ricardian authors ever envisioned an actual purchase and sale of used fixed capital goods before and after every production period. Sraffa himself highlights that the problem is *purely technical*, that fixed capital prices derived in the joint production method are "*only book values*" (Sraffa, 1960, p. 64). Just as in the transfer of value approach, they do not represent any actual market spot prices. Schefold (2011, pp. 182, 189), who of course applies the joint production method himself discusses the issue more extensively than others and calls secondhand markets for capital goods "inevitably missing" and "almost absent from this world". Deprez (1990, p. 59) points out that the idea of an actual representation of true market transactions would imply a full set of secondhand spot markets for fixed capital which is (unlike markets for finished commodities including new fixed capital) not a necessary operating requirement for the capitalist system. Besides this logical issue, he also refers to the empirical problems of a lack of standardization of fixed capital, the non-existence of placement markets and the observation that fixed capital secondhand sales usually only occur in bankruptcy. As an actual representation of a literal real economic process, the joint production approach

[9] In some case there was no detailed formal representation.

would be absurdly implying a *sale of all assets of all firms every year, month or day* (depending on the production period length). While authors critical to the joint production approach like Moseley (2009, 2011) correctly state that used machines are generally not sold on the market, the nature of the Sraffian equations is misinterpreted as a representation of an actual transaction process. They are purely imagined accounting transactions which do not require real equivalents to be valid concepts. The popularity of the combination of joint production with annual profit rate consistency stems exclusively from the ease and simplicity with which they allow to treat fixed capital in an equation system. Establishing the exclusively technical nature of the joint production versus transfer of value question, of course, does not automatically imply unimportance. However, it is entirely irrelevant for the following discussion of the economic problems of depreciation and the mathematical solutions offered further below. This should always be kept in mind in this article.

B. *First Economic Problem: Fulfilling All Three Price Principles*

Neoclassical equilibrium prices balance supply and demand conditions in a specific moment in time. Prices reflect social scarcity by depending on the composition and distribution of resource endowments, technology and utility functions.

Natural prices or prices of production are the equilibrium prices in Classical Political Economy. They represent a long-period equilibrium and act as centers of gravity for actual observable short-period market prices. Only the latter balance supply and demand while they fluctuate around the natural price centers. Without fixed capital, prices of production can be represented by the price vector $\boldsymbol{p}$ that solves

$$w\boldsymbol{l} + (1 + r^*)\boldsymbol{pA} = \boldsymbol{p_b b} \tag{5}$$

where $\boldsymbol{l}$ is the labor input requirement vector, $\boldsymbol{A}$ the matrix of material input requirements, $\boldsymbol{b}$ the output vector, and all other variables as defined above. This simple circulating capital model equation can be used to illustrate several important principles of price theory that Classical economists seek to establish.

The first one is that prices of production should be unique in the sense that the law of one price holds. This principle is also accepted in Neoclassical theory. In each of the existing solutions to the fixed capital problems, competition enforces identical prices. Besides small and obvious deviations due to taxes or transportation costs, there is no theoretical reason for significant and persistent systematic differences for identical commodities in a long run equilibrium. A homogeneous product produced with machines of different vintages must command identical prices.

Second, prices of production require that profit rates are equalized between industries and normalized to the "natural", "normal" or "average" rate of return (r^* in equation 5). This is different in Neoclassical theory, where profit rates calculated on production costs are not equalized. Instead, they are quasi-rents on different capital good types and reflect their individual scarcities. However, besides this difference, the principle of profit rate equalization is not as absolute in Classical economics as it may seem. Classical theory recognizes firm heterogeneity and profitability differentials at every snap-shot point in time for each industry, firm and investment. Authors also describe natural disequalization mechanisms for profit rates within industries (that result from the pursuit of surplus profit) in regularly functioning competition (Shaikh, 2008). It can even be shown that profitability differentials of individual firms can persist or diverge throughout limited periods precisely because of fixed capital characteristics (Keil, 2016). Here, these complications are avoided with the high level of abstraction of the prices of production analysis. However, when fixed capital is used, there is one open question remaining: which profit rates are to be equalized? The usual answer is that the set of all individual annual rates of return are considered. The reason why this should be done seems to be more that it is technically convenient to have only one profit rate on all capital goods in a production equation than to have the same profit rate in every different process that uses different vintages of capital goods. However, it is logically unclear why profit rates on different vintages of the same individual machine should equalize. Unlike in the case of competition and interindustrial capital reallocations that equalize profit rate differentials between industries, such a mechanism or an equivalent to it is not existent. Capital goods are not sold after every year, month, week or day on a secondary market (Moseley, 2011). The innovation here is to acknowledge what capitalists really do and what corporate finance already realized decades ago: when an investor considers buying a productive asset, he or she first estimates the cash flows that can be derived from the asset *throughout its entire economic lifetime.* Second, he or she applies the net present value or the *internal rate of return* criterion.[10] If the internal rate is above the hurdle rate, the investment will be undertaken. Here, this means that in equilibrium the internal rate of return must be equal to the normal rate. While this might be seen as a weaker version of the profit rate normalization principle (compared to the normalization of all individual annual profit rates), it is really a more realistic description of what capitalists actually look at.[11]

Third, natural prices make the (simple or expanded) reproduction of the economic system possible by allowing to cover costs of production. From the perspective of the capitalist, this represents the wage bill ($w\boldsymbol{l}$ in equation 5) and the expenses for material input requirements ($\boldsymbol{pA}$). Thus, without fixed capital (and true joint production) the definition of "cost proportionality" is trivial. With fixed capital it becomes more complicated. Like the second principle, it can be interpreted in different ways: a) prices are proportional to "costs of production", or the value of all inputs used in the lowest-cost production process

[10] They are in simple and standard cases equivalent decision criteria as the internal rate of return is the discount rate that produces a net present value of zero.

[11] Of course, individual annual profits do actually play a role when efficiency, output and costs of a machine change and a truncation decision must be made. However, the assumptions here abstract from that.

that is generally reproducible (plus a markup that allows to obtain the normal profit rate);[12] b) prices are derived from the "conditions of production". This describes more generally the requirements for the reproduction of the economic processes and may or may not only include costs. Both interpretations reflect the Classical agenda to derive prices from reproductive conditions of the economic system. Costs and purely objective factors from the sphere of production are used as opposed to the Neoclassical attempt to find subjective prices based on utility, scarcity and exchange in the sphere of distribution. Taken together, prices that simultaneously include the markup that yields a normal profit rate *and* a charge that covers costs, implies Classical, not Neoclassical equilibrium prices.[13]

The exact definitions of "costs of production" mentioned is vague, but very essential here. With purely circulating capital, no complication occurs, since all purchased inputs are used up on the production period. Fixed capital, however, is used up during a time period that covers multiple periods of production. It is clear that *the sum of fixed capital unit costs or depreciation charges on all the output units produced during this asset lifetime must add up to the initial purchase price of the asset*:

$$K_1 = \sum_{t=1}^{T} D_t \tag{6}$$

[12]The view that prices are related to or regulated by labor values, for example such that deviations are small and predictable, is a version of this interpretation.

[13]Schefold (1989, p. 181) dismisses the alternative narrower "cost" concept in favor of this broader one on the basis of two arguments: 1. the interdependence of prices of basics in single-product systems and the dependence of a commodity's price on the profit rate; 2. the dependence of the price of products of joint production processes on costs *and* their use as means of production. While there is no question about the existence of these two phenomena it is not apparent why the first one is at odds with the "costs of production" principle. There is no contradiction in the fact that costs can be derived by means of solving a simultaneous equation system and that the price of a commodity depends on its own price and the profit rate. The latter is a cost if charged by a supplier. As a mark-up (if charged by the capitalist in question himself) it just means that price and costs are not identical – but they are still proportional to each other. The second problem can indeed be answered by substituting the stronger and more concrete concept of "costs" for the weaker and more general one of "conditions" of (re)production. However, it can also be responded to by admitting that joint production represents a weakness for the Classical equilibrium concept. The stronger, more concrete criterion does not hold, while the weaker, more general version does. This does not imply that the stronger one should be completely abandoned. Neoclassical equilibrium concepts suffer from much worse problems and a stronger and more demanding concept like the Walrasian equilibrium is, for example, not abandoned in favor of the less demanding and weaker trading equilibrium.

This is the "weak" and minimum requirement of a cost consistent determination of depreciation charges. A stronger version suggested here is that the price of output produced in every single year must be proportional to its production period costs. But how should the total costs given by the initial purchase price of the fixed asset be distributed among years and output? Given the assumptions (constant efficiency, physical wear-and-tear as the only factor that consumes a machine, constant maintenance and output), the same quantity of fixed capital is used-up in the actual production process in every year when measured in any possible physical way (see e.g. Marx (1990, pp. 272-273) on the factors that determine depreciation). Most intuitive is the annual reduction in the number of output units that can be produced with the application of the asset in total (the reduction in years of economic life available is equivalent here). This reduction is due to some type of physical deterioration or weakening of the asset that reduces the ability of the a machine's material substance to sustain X number of production processes less than it could in the previous period. This physical change is neither contradicting the assumption that it has no negative effects on productivity, nor that it might be invisible and only manifest itself in a spontaneous "implosion" of the asset at the end of the final period. However, the only cost consistent way to determine the corresponding monetary use-up of the fixed asset is one where *the annual depreciation charge is proportional to the annual physical use-up of the fixed asset*. This is the stronger definition of cost consistency used here. Given the assumptions used here, it implies linear depreciation

$$D_t = D = K_1 \frac{1}{T} \tag{7}$$

since $\frac{1}{T}$ is the percentage reduction in asset lifetime (or number of output units producible) and K_1 the nominal price of the original fixed asset.

The analysis in this subsection suggests that it is possible to formulate the three core principles with an eye on fixed capital problems, such that "strong" and “weak” versions exist for the second and third. In the strong versions, they describe that throughout the lifetime of fixed capital, there must be:

1) the law of one price: identical equilibrium prices for all output units;

2. “annual profit rate consistency”: identical annual profit rates are equalized with each other and with the normal rate;

3. “annual cost consistency”: identical annual fixed capital depreciation costs, proportional to the physical use-up of the asset.

The “weak” versions of latter two principles are, that throughout the lifetime of a fixed asset, there must be:

2b. “lifetime profit rate consistency”: the internal rate of return on investments in fixed capital obtained throughout its lifetime must equal the normal profit rate;

3b. “lifetime cost consistency”: the sum of depreciation costs that accrue throughout the asset lifetime must equal the price of the original newly produced fixed asset.

Note that annual profit rate (or cost) consistency also always implies lifetime profit rate (or cost) consistency. The other way around, this is generally not true whenever the profit rate is greater than zero (implying that the description of the former as “strong” and the latter as “weak” is a sensible one). For these cases, the inclusion of fixed capital makes it mathematically impossible to fulfill conditions 1, 2 and 3 (both strong definitions) at the same time, while it is always possible to either fulfill 1, 2 and 3b or 1, 2b and 3. Conditions 1, 2 and 3b are generally accepted by Neo-Ricardians. Conditions 2b and 3 are proposed here as equivalent and logically at least equally valid counterparts.

C. Second Economic Problem: Physical or Monetary Capital?

The second economic problem involves the choice of the profit rate that prices of production are supposed to normalize: What is the capital stock from which one shall compute the annual profit rates or the internal rate of return? Sraffa and all Neo-Ricardians point to the value that is contained in the material assets invested in the industry, implying a "physical capital" concept. The value of this physical fixed capital is equal to its initial price, minus the sum of all annual depreciation charges accrued to the respective asset age. Opposed to that, Marx describes a "monetary capital" concept, which includes this physical component and the "accumulated depreciation fund". The fund is the sum of all accrued depreciation charges. Together, they always add up to the initial purchase price of the asset. This monetary capital concept can have different possible versions, which depend on the ability of the depreciation fund to obtain an interest rate. While it is not always apparent which exact concept is used by the Classical authors, Marx (1992, p. 261) clearly points towards this capital concept. He also implies that the treatment of the depreciation fund is dependent on the development of the credit system (and on the level of abstraction that is seen as useful for a certain analysis). For the interest rate i obtained by the depreciation fund, this suggests three different possibilities: it pays either no return at all, one that is equal to the normal profit rate r^*, or one that is different from (and smaller than) the normal profit rate. Thus, the monetary concept of capital allows for a treatment of the interest rate that is separate and different from the profit rate. This yields a more differentiated and realistic analysis. It is closer to Marx's view that the profit rate is the upper limit of the interest rate and that it adds a "profit of the enterprise" to it (Panico, 1988, pp. 49, 53). It also seems somewhat more realistic than the physical view with respect to the fact that a true "reproductive condition" is that the fixed capital asset must be replaced at the end of its lifetime. Only when enough funds have been accumulating until that point, is this replacement and the repetition of the whole production process possible. Even the physical approach should acknowledge this at least implicitly. An approach that analyzes capitalism as it really operates should analyze this problem properly and account for it explicitly.

III. "Correct" Solutions to the Fixed Capital Problems

A. *Physical Capital & Annual Profit Rate Consistency*

The Sraffian solution relies on the "physical" concept of capital while the computation of the amortization component M (that includes depreciation and mark-up charges for the return on fixed capital) is annual profit rate consistent. For profit rates on the shrinking physical fixed asset value to remain constant throughout the lifetime, the profit charges must decline correspondingly. Thus, depreciation increases in time to keep amortization constant – ceasing to be proportional to the actual use-up of fixed capital in the production process in any production period. It is not a function of the use-up process of the physical substance of the fixed capital, but the profit rate and the age of the machine. The price of capital in every year K_t equals the present value of all future annual amortization charges or cash flows, M, remaining in the life time of the fixed asset. M is found with the standard annuity loan formula used by Ricardo (1951, pp. 55-54, 57-60) in one section in the first edition of the Principles. It was also applied by Bortkiewicz (1907) and (Sraffa, 1960, §75). The latter also called it the "accountant's method" (Kurz and Salvadori, 2005, p. 500). The approach was even known to earlier economists like Florencourt in 1781 and scholastic and cameralistic authors (Schefold, 2011, pp. 182, 188). The formulas for M are

$$M = K_1 \frac{r^*[1+r^*]^T}{[1+r^*]^T - 1} \tag{8}$$

and for K_t

$$K_t = M \frac{[1+r^*]^T - 1}{r^*[1+r^*]^T} \tag{9}$$

The annual total depreciation charge in year t, D_t, is determined by equation 3 while the total profit on fixed capital or mark-up charge Π_t in year t can easily be obtained as

$$\Pi_t = M - D_t \tag{10}$$

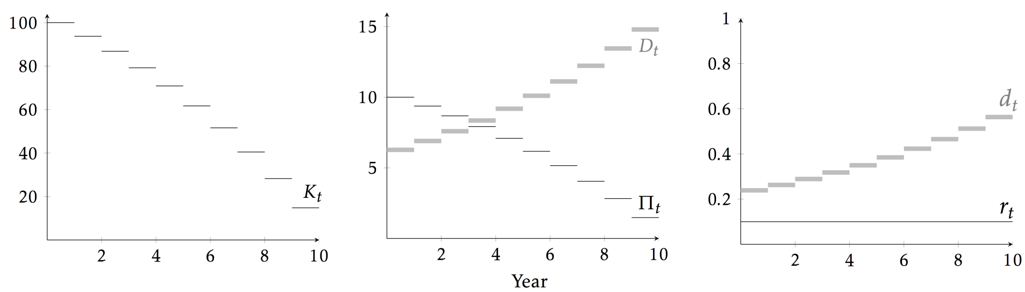
The annual profit rate is computed as

$$r_t = \frac{\Pi_t}{K_t} \tag{11}$$

As noted, the application of the annuity formulas implies that $r_t = r^* \ \forall \ t$.

Figure 1 illustrates key variables for the case of an industry that uses only a fixed capital asset $K_1 = 100$ with a life time of 10 years, constant output of 10, total variable costs of 10 and a normal profit rate of 10%. d_t are depreciation costs as a share of the unit price. π_t. M equals 16.275, unit fixed capital costs $m = \frac{M}{output} = \frac{16.275}{10} = 1.6275$ and the price is 2.6275. One can see the problem that depreciation rates vary by a factor of more than two (between 0.26 in the first and 0.62 in the last year).

Figure 1: Physical capital & annual profit consistency



K_t: physical capital, D_t: depreciation, Π_t: profit on fixed capital, $d_t = \frac{D_t}{price}$, $r_t = \frac{\Pi_t}{K_t}$.

Noting that in the Sraffian solution $\Pi_t = r^* K_t$ and using the numbers from this numeric example implies that the joint production equation 2 has 10 versions, one for each possible fixed asset vintage:[14]

$$
\begin{aligned}
10 + 100 + 10 &= 2.627 * 10 + 93.73 && (t = 1)\\
&\vdots\\
10 + 61.69 + 6.17 &= 2.627 * 10 + 51.59 && (t = 5)\\
&\vdots\\
10 + 14.795 + 1.48 &= 2.627 * 10 + 0 && (t = 10)
\end{aligned}
$$

B. *Physical Capital & Annual Cost Consistency*

A wide variety of depreciation mechanisms is described by Marx (1990, pp. 272-273): the use-up of fixed capital due to the intensity to wear and tear during the production process; the wear and tear due to elements of nature and time; the "moral depreciation" where fixed capital loses value because of a normal path of technical change that lowers costs of production and raises effectiveness of machinery. Under the strict equilibrium assumptions used here (constant capacity utilization, constant technical change), simplified versions of each of these phenomena can be approximated through linear depreciation. This is explicitly found in Torrens (1818, pp. 337)[15] and in Marx (1992, pp. 255, 263, 530 ff.). Some current Marxist scholars also point towards this approach (Shaikh, 1980; Moseley, 2009).

Linear depreciation derives the price of fixed capital from past costs of production and the actual physical use-up of the underlying asset described above (while the common method derives it from the present value of future income streams). The identical depreciation charges are given by equation 7 and the value of the physical fixed capital of vintage t is simply

$$K_t = K_1 - tD \tag{12}$$

[14]The transfer of value equations are omitted here but can be obtained in an obvious way.

[15]"Suppose that a tenth of these fixed capitals is annually consumed, and that the rate of profit is 10 per cent. then, ... his fixed capital has been reduced by the process of production from L. 1500 to L. 1350, ..."

However, different versions of how linear depreciation is treated can differ with respect to the computation of the (now constant) mark-up or profit component Π in the amortization charge M:

$$\Pi = M - D \tag{13}$$

With physical capital, only the value of the remaining capital after depreciation is considered as the base upon which profit is computed. This means that, since Π is invariant through time (equation 13), the annual profit rate is growing with the decrease in value of fixed capital invested.

To evaluate this depreciation method, one can compute the internal rate of return on the investment in the fixed capital asset. It has the interpretation of the lifetime profit rate $\tilde{r}$ on fixed capital that costs K_1 and earns M annual payments over T years. It can be obtained by solving the standard annuity bond formula

$$K_1 = \sum_{t=1}^{T} \frac{M}{(1+\tilde{r})^t} = M \frac{(1+\tilde{r})^T - 1}{\tilde{r}(1+\tilde{r})^T} \tag{14}$$

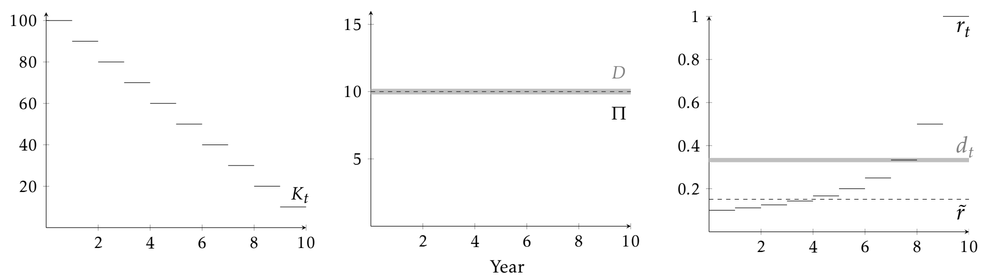
for the discount rate $\tilde{r}$. This requires the application of a root solving procedure like the Newton-Raphson method whenever $T > 2$. Note that for the computation of the internal rate of return, M is treated in this physical view as a cash flow because it is directly available and does not need to be retained in a depreciation fund (see next subsection). This is, of course, exactly the same in the common method discussed in the previous section.

The open question is how to determine Π. In the common representation of linear depreciation also found in Torrens (1818, p. 337) and Marx (1992, p. 255) the value of a new machine appears to be used for the computation of the profit charge by just multiplying the normal profit rate with it:

$$\Pi = r^* K_1 \tag{15}$$

Using the example also applied in the previous section, this implies that the annual profit rates rise from 10% in the first to 100% in the last year. Figure 2 illustrates this. The price is now 3 and depreciation accounts for one third of it, $M = 20$ and $m = 2$. It also shows that $\tilde{r} = 15.1\%$, implying the obviously inconsistent result that one can obtain a surplus profit on an investment in fixed capital. Only the first year profit rate equals the normal rate. The ones of all other years and the internal rate of return differ from it.

Figure 2: Physical capital & profit rate inconsistency



K_t: physical capital, D: depreciation, Π: profit on fixed capital, $d_t = \frac{D}{price}$, $r_t = \frac{\Pi}{K_t}$, $\tilde{r}$: internal rate of return on fixed capital.

The alternative is to set Π such that $\tilde{r} = r^*$. For this one can rearrange equation 14 for M:

$$M = K_1 \frac{\tilde{r}(1+\tilde{r})^T}{(1+\tilde{r})^T - 1} \tag{16}$$

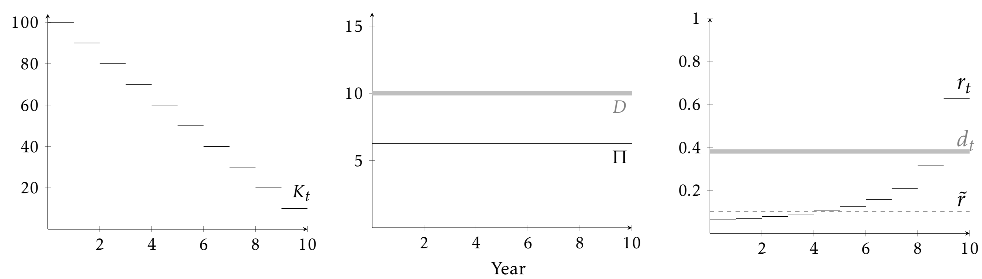
Inserting equation 16 into 13 allows to solve for Π. Setting $\tilde{r} = r^*$ yields the "correct" profit charge:

$$\Pi = K_1 \frac{r^*(1+r^*)^T}{(1+r^*)^T - 1} - D \tag{17}$$

This produces a linear depreciation pattern that is not just cost consistent on an annual, but also profit rate consistent on a lifetime basis.

Continuing with the example numbers, Π now becomes 6.275 which corresponds to $M = 16.275$, $m = 1.6275$ and a price of production of 2.6275. The latter is the identical value from the common method. However, figure 3 shows how the method differs with respect to the valuation of fixed capital. It depreciates linearly unlike in the common method. The fixed capital cost becomes strongly consistent on an annual basis and not just throughout the asset lifetime. Annual profit rates are disequalized while the internal rate of return equals the normal rate, $\tilde{r} = r^*$.

Figure 3: Physical Capital & annual cost consistency



K_t: physical capital, D: depreciation, Π: profit on fixed capital, $d_t = \frac{D}{price}$, $r_t = \frac{\Pi}{K_t}$, $\tilde{r}$: internal rate of return on fixed capital.

This method requires the adjustment of the joint production and transfer of value pricing equations 2 and 4 to:

$$wl + (1 + r_t)K_t = p_b b + K_{t+1} \tag{18}$$

and

$$wl + r_t K_t + D_t = p_b b \tag{19}$$

where $r_t K_t = \Pi$ ensures that the law of one price is not violated. Similarly, Shaikh (2015, Appendix 6.4) shows how the linear depreciation method can be represented in a Sraffian system when a monetary concept of capital and zero interest on the depreciation account is assumed. The equations here are more general as they allow to represent also a physical concept and annual profit rate as well as annual cost consistent solutions for all possible interest rates (see below). The corresponding 10 versions of the joint production equation for the 10 vintages become:

$$
\begin{aligned}
10 + 100 + 6.275 &= 2.6275 * 10 + 90 && (t = 1)\\
&\vdots\\
10 + 60 + 6.275 &= 2.6275 * 10 + 50 && (t = 5)\\
&\vdots\\
10 + 10 + 6.275 &= 2.6275 * 10 + 0 && (t = 10)
\end{aligned}
$$

C. Monetary Capital & Annual Cost Consistency

The "monetary" understanding of capital is also present in Classical Political Economy. Marx (1992, 1990) spends maybe the greatest effort on emphasizing that fixed capital is not just a physical, but most importantly also a monetary category. An implication for him is that it gradually changes form into what he calls the "money reserve fund", "amortization fund" or "accumulated money fund" (Marx, 1992, pp. 251, 261, 529). The recognition of this (see also Shaikh, 2015) means that the base for the computation of profit rates is not the remaining value of the fixed capital invested in a machine only. It must also include the accumulated depreciation fund, F_t.[16] Depending on further assumptions about the interest rate obtained on it this can make a difference for annual amortization charges and prices of production. Thus, it becomes clear why the "financial" capital should be included in the calculations of the capitalist. If fixed capital forces capitalists to keep a fraction of their capital from being invested at the normal profit rate (like on an actual savings account that yields a lower return in the form of the interest rate), then this must be considered initially when the investment is made. If one would not take account of this, them the total return on a different investment that uses less fixed capital would be higher. Abstracting

[16] "Accumulated depreciation fund" is a more precise terminology since only the depreciation component is retained and not the entire amortization charge.

from any possible differences in risk, this cannot be an equilibrium solution since the overall returns on investment are not equalized and differ from the normal rate of return.

F_t always grows each year exactly by the amount with which the value of physical fixed capital shrinks. With linear depreciation it can be easily represented as

$$F_t = tD \tag{20}$$

Total capital κ stays constant through time:

$$\kappa = K_t + F_t \quad \forall\ t \tag{21}$$

Since interest on the depreciation fund is immediately available for free disposal such as new investment, it should be viewed as a profit income category.[17] Thus, the total mass of annual profit, π_t, is now variable and grows in t if $i > 0$

$$\pi_t = \Pi + iF_{t-1} = \Pi + i(t-1)D \tag{22}$$

Note the assumption that depreciation accrues and changes the capital stock composition at the end of a period. In this case the interest rate charged in year t is computed on F_{t-1} (which was present at the beginning of the year), not on F_t. The annual profit rate can accordingly be written as

$$r_t = \frac{\Pi + iF_{t-1}}{\kappa} = \frac{\pi_t}{\kappa} \tag{23}$$

It is important to keep in mind that only Π and not iF_{t-1} is part of the price of production. The latter will never occur in any of the joint production and transfer of value pricing equations 2 and 4 (or 18 and 19) as these proceeds are

[17] An alternative treatment of monetary capital is discussed in the appendix and shown to be less intuitive and more complicated.

obtained through the financial system, lending or saving accounts, and not from production and products sold.

The assumption of depreciation charges being paid into a fund that must be held permanently until the asset is fully depreciated has implications for the computation of the internal rate of return. In the physical approach, each D or D_t is treated as a cash flow together with Π or Π_t. In the monetary approach, D is not representing a cash flow since it stays "tied-up" in the assets of the firm and cannot be spent or reinvested arbitrarily. One balance sheet asset position becomes another one that is equally "fixed" with respect to economic reproduction. On the other hand, their aggregate in the final period T, $F_T = K_1$ is such a cash flow and equivalent to each D_t in the physicalist Sraffian approach. F_T can be spent on a new fixed capital asset of the same type. The whole series of cash flows which can be obtained by investing in K_1 can then be expressed in a slightly more complex way as

$$K_1 = \Pi \frac{(1+\tilde{r})^T - 1}{\tilde{r}(1+\tilde{r})^T} + \frac{iD}{\tilde{r}} \left(\frac{(1+\tilde{r})^T - 1}{\tilde{r}(1+\tilde{r})^T} - \frac{T}{(1+\tilde{r})^T} \right) + \frac{K_1}{(1+\tilde{r})^T} \tag{24}$$

$\tilde{r}$ is again the internal rate of return. The first term represents the present value of the T annual payments of Π (the value of an annuity bond). The second term represents the present value of the $T-1$ interest payments on the depreciation account at an interest rate of i. The third term is the present value of the final access to F_T (the value of a zero bond). Terms one and three taken together form the valuation formula of a coupon bond. Terms one and two represent the also common formula for the present value of an arithmetically changing annuity bond. The derivation can be reviewed in standard financial mathematical textbooks (see Tietze, 2009, pp. 149).

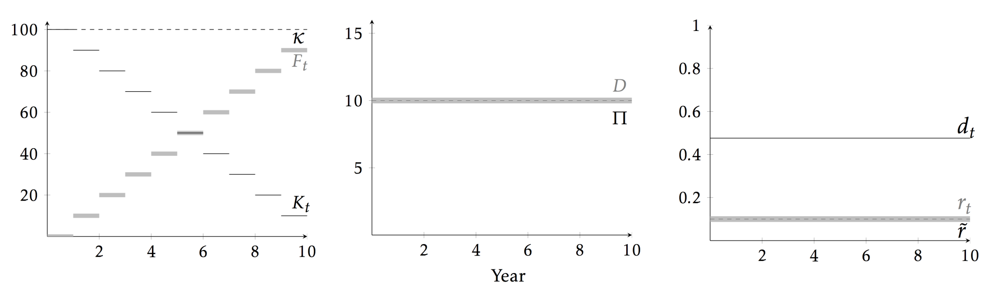
The "correct" Π can then be found by rearranging equation 24 and substituting

$\tilde{r} = r^*$:

$$\Pi = r^* K_1 - iD\left(\frac{1}{r^*} - \frac{T}{(1+r^*)^T - 1}\right) \tag{25}$$

Assume F_t earns no interest ($i = 0$). This corresponds to an underdeveloped credit system or to a simplifying abstraction. It causes the third term on the right hand side of equations 24 and 25 to drop out, producing equation 15. For the numbers from the example, $\Pi = 10$. This is illustrated in the graph below. The price of production in this case is 3, as in the "incorrect" case where no depreciation fund is assumed. This shows it is possible, and even likely, that Torrens (1818, p. 337) and Marx (1992, p. 255) assume precisely this monetary case with the particular assumption made here. In this case, the depreciation method that appears inconsistent if one looks at it from a perspective centered on physical capital and annual profit rate consistency (Gehrke, 2011; Lager, 2006; Kurz and Salvadori, 1997) becomes strongly, annually consistent with respect to profit and costs when written in a monetary way (see figure 4). It is not necessary to know the aggregate fixed capital quantity invested in the industry to determine the profit charge (as suggested by Moseley (2011)). The latter is simply found by setting the internal rate of return equal to the normal profit rate and applying the correct annuity formula.

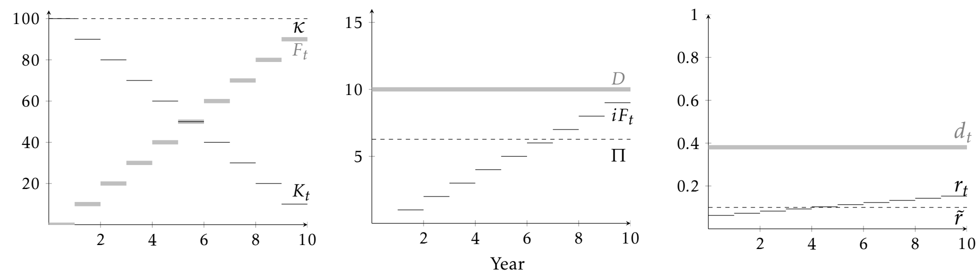
Figure 4: Monetary capital & annual cost consistency ($i = 0$)



K_t: physical capital, F_t: depreciation fund, $\kappa = K_t + F_t$, D: depreciation, Π: profit on fixed capital, $d_t = \frac{D}{price}$, $r_t = \frac{\Pi}{\kappa}$, $\tilde{r}$: internal rate of return on fixed capital.

Assume that F_t earns the general rate of return ($i = r^* = 0.1$). This corresponds to a highly developed or perfect credit system and is realistic if, for example, the fund can be used for risky short term investments or internal purchases of circulating capital in a large company.[18] Using equation 25 and all other example data from before implies a "correct" Π of 6.2745 and an equilibrium price of 2.6275. Figure 5 illustrates this.

Figure 5: Monetary capital & annual cost consistency ($i = r^*$)



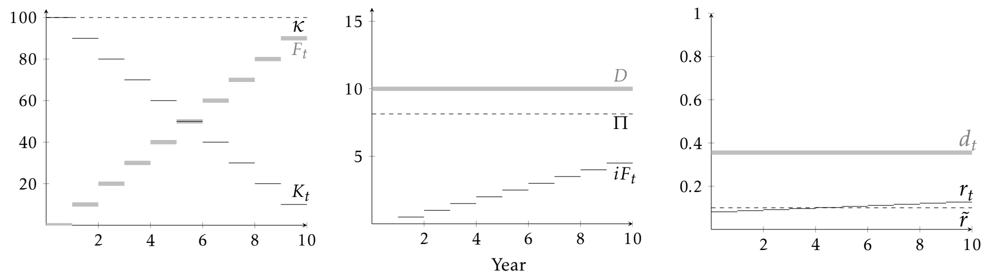
K_t: physical capital, F_t: depreciation fund, $\kappa = K_t + F_t$, D: depreciation, Π: profit on fixed capital, iF_t: interest on F_t, $d_t = \frac{D}{price}$, $r_t = \frac{\Pi + iF_{t-1}}{\kappa}$, $\tilde{r}$: internal rate of return on fixed capital.

Finally, assume that F_t earns an interest rate that is smaller than the normal profit rate ($i < r^*$). This may be the most realistic case. It corresponds to a more developed but not perfect credit system where capitalists lend out savings to other capitalists or place them as bank deposits. Using equation 25 and setting $i = 0.05$ the "correct" Π becomes 8.1373 and the price of production changes to 2.8137 (see figure 6). The higher numbers make sense as the interest earned on the depreciation account is lower and needs to be compensated for in order to obtain the same normal rate of return.

Since the accumulated depreciation fund and the interest obtained on it are not part of the prices of production, the price equations 18 and 19 do not need to be adjusted. In the case where $i = 0.05$ and using the example numbers, the corresponding 10 versions of the joint production equation 2 for the respective 10

[18]The common annual profitability consistent method involves the same assumptions about the degree of development of the credit system or even further reaching ones (full resale markets for fixed assets).

Figure 6: Monetary capital & annual cost consistency ($i < r^*$)



K_t: physical capital, F_t: depreciation fund, $\kappa = K_t + F_t$, D: depreciation, Π: profit on fixed capital, iF_t: interest on F_t, $d_t = \frac{D}{price}$, $r_t = \frac{\Pi + iF_{t-1}}{\kappa}$, $\tilde{r}$: internal rate of return on fixed capital.

vintages are:

$$10 + 100 + 8.1373 = 2.8137 * 10 + 90 \qquad (t = 1)$$
$$\vdots$$
$$10 + 60 + 8.1373 = 2.8137 * 10 + 50 \qquad (t = 5)$$
$$\vdots$$
$$10 + 10 + 8.1373 = 2.8137 * 10 + 0 \qquad (t = 10)$$

D. *Monetary Capital & Annual Profit Rate Consistency*

In its normal form described above, the common annual profit rate consistent method is based on a physical concept of capital. But as in the case of the linear method, one can also apply a monetary definition.

Unlike all the other cases previously analyzed, where regular and arithmetically changing annuities could be used, there is no standard financial formula to find M and the values of K_t ($\forall\ 0 > t < T$).

Since the monetary capital stock is constant (equation 21) and must be used to compute the annual profit rate on total fixed capital, the total annual profit mass π_t must also be constant in order to have a constant annual profit rate:

$$\pi = \Pi_t + iF_{t-1} = r_t K_t + i(K_1 - K_{t-1}) \tag{26}$$

Since iF_{t-1} increases with the growing depreciation fund, the profit charge Π_t must decrease correspondingly. We know that in the first year, there is no depreciation fund, but the annual profit rate on total capital must still be the normal rate. Thus,

$$\pi_1 = \pi = r^* K_1 \tag{27}$$

K_0 is given by costs of production and $K_T = 0$. Using the numeric example, the remaining capital stock values can be found from the system of $T-1$ equations:

$$\begin{gathered} r_2 K_2 + i(K_1 - K_2) = r^* K_1 \\ r_3 K_3 + i(K_1 - K_3) = r^* K_1 \\ \vdots \\ r_T K_T + i(K_1 - K_T) = r^* K_1 \end{gathered}$$

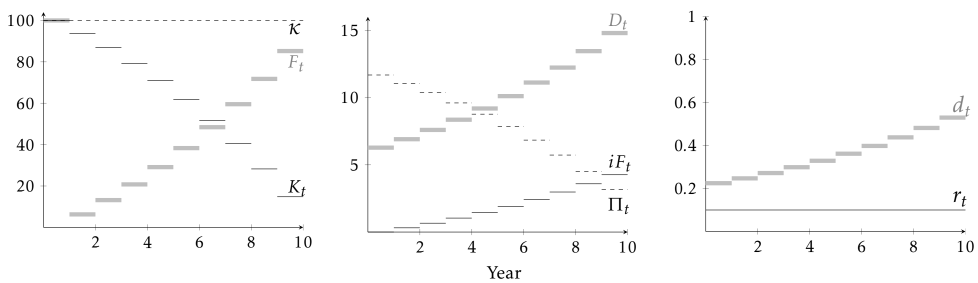
The $T-1$ equations have $T-1$ unknown K's ($K_2 - K_{T-1}$) and $T-1$ unknown r's ($r_2 - r_T$). Knowing that $\Pi_t + D_t = r_t K_t + K_t - K_{t-1} = M$ adds T more equations:

$$\begin{gathered} r^* K_1 + (K_1 - K_2) = M \\ r_2 K_2 + (K_2 - K_3) = M \\ \vdots \\ r_T K_T + (K_T - 0) = M \end{gathered}$$

which only adds the unknown M. The result is a system with $2 * T - 1$ equations and unknowns. Thus, the system is solvable and yields $M = 17.95$ and $p = 2.795$ for $i = 0.05$. The numbers for $i = 0.1$ are $M = 16.2745$ and $p = 2.6275$, which is identical to the two solutions described for the physical capital. For $i = 0$, M, Π_t, D_t and the price become identical to the linear monetary method when it sets

$i = 0$. For the sake of conserving space, only $i = 0.05$ is visualized in figure 7.

Figure 7: Monetary capital & annual profitability consistency ($i < r^*$)



K_t: physical capital, F_t: depreciation fund, $\kappa = K_t + F_t$, D_t: depreciation, Π_t: profit on fixed capital, iF_t: interest on F_t, $d_t = \frac{D_t}{price}$, $r_t = \frac{\Pi_t + iF_{t-1}}{\kappa}$.

$$10 + 100 + 10 = 2.6275 * 10 + 93.73 \qquad (t = 1)$$
$$\vdots$$
$$10 + 70.88 + 7.09 = 2.6275 * 10 + 61.69 \qquad (t = 5)$$
$$\vdots$$
$$10 + 14.8 + 1.48 = 2.6275 * 10 + 0 \qquad (t = 10)$$

E. Interpretation and Evaluation

The use of the numerical example in the alternative solutions presented here, illustrates two important implications of the different depreciation methods and financial formulas:

First, under a monetary concept of capital, the lower the interest rate on the depreciation fund, the higher the profit markup and the price of production. This explains a possible counterintuitive scenario where an increase of the interest rate (which is not affecting the normal profit rate) *lowers* the relative prices of goods produced with a greater amount of fixed capital. This finding is surprising, unless, of course, the industry average leverage ratio differs systematically from those of other industries.

Second, it is not only the case that any change in the assumed interest rate on the depreciation account impacts the prices (see table 1). When the interest rate on the depreciation account is smaller than the normal rate, then a switch from a physical to a monetary capital concept changes the prices of production. If the interest rate is between the normal profit rate and zero, then there is a difference between the prices implied by the annual profit rate consistent and the annual cost consistent method. Thus, Sraffa's intuition that different "correct" depreciation schemes just affect the division of M, does not hold when a monetary concept of capital is used and an interest rate is applied that is greater than zero. The effect on actual equilibrium prices shows that the question of choosing the correct method is not just some kind of esoteric question and accounting problem.

Table 1: Summary of all "correct" solutions to the treatment of fixed capital

Capital Concept:	"Physical"			"Monetary"	
"Annual Consistency":	Profit	Cost		Profit	Cost
Constant Annual Depreciation	no	yes	$i = 0$	yes	yes
			$i = 0.05$	no	yes
			$i = 0.1$	no	yes
Total Depreciation = initial fixed capital price	yes	yes	$\forall\ i$	yes	yes
Equal annual profit rates	yes	no	$i = 0$	yes	yes
			$i = 0.05$	yes	no
			$i = 0.1$	yes	no
Internal rate of return = normal profit rate	yes	yes	$\forall\ i$	yes	yes
Formula to find amortization	annuity bond	annuity bond	$i = 0$	coupon bond	coupon bond
			$i = 0.05$	linear equation system	arithmetically changing annuity + zero bond
			$i = 0.1$	linear equation system	arithmetically changing annuity + zero bond
Price of Production	2.6275	2.6275	$i = 0$	3	3
			$i = 0.05$	2.795	2.8137
			$i = 0.1$	2.6275	2.6275

The table summarizes the different approaches to the treatment of fixed capital. Capital is either "physical" (only the value of real assets remaining after accumulated depreciation) or "monetary" (physical plus the accumulated depreciation fund). Depreciation can either be cost or profit rate consistent on an annual basis. The interest rate i on the depreciation fund is either equal to the normal rate (10% in the example in the text), zero, or between zero and the normal rate (5% are used as an example). All solutions offered are lifetime consistent with respect to costs or profitability. Prices of productions are dependent on the interest rate.

Another interesting finding is that with constant efficiency there is no truncation problem that must be solved, as r_t is either constant or increasing for any of the cases considered. The analysis of changing efficiency over the lifetime of a machine is an important issue and would lead to truncation problems. However, due to space limitations, this must be the subject of future research.

One complication must be discussed here. The linear depreciation approach might appear to be inconsistent, in that, *if* used fixed capital is sold (i.e. at $t > 0$) then it will not be bought for current book value implied by the method. It will be sold at a price that yields the normal profit rate as an internal rate of return. Falsely, this problem seems to not occur in the common method when the machine or asset is sold since the price of capital is always equal to the future cash flows discounted at the normal rate. However, this is *only* true for a transfer within an industry (a re-use of the asset in an identical production process). The very same problem occurs in the common method if the system is "interlocked": once the same fixed capital asset is used in a different production process, it is generally safe to assume that the intensity and lifetime will differ. If this is the case and a machine is sold from one sector to another, then the price the buyer is willing to pay is always different from the book value of the machine. This holds for *any* method of treating fixed capital. It follows that the seeming weakness of the linear depreciation method is also existent for the common method in any realistic and more general model. Transfers of assets between industries and applications are always possible for most fixed assets. Using the example values from above, the common Sraffian solution implies $K_4 = 70.9$ and the linear method $K_4 = 60$. Assume the same asset is used in another production process at twice the intensity and could accordingly be used for another 3 years (no other difference is assumed). If that vintage of a machine is transferred, the price a capitalist in this industry is willing to pay must yield the normal profit rate. This price found using Sraffa's annuity formula and turns out to be 65.6. Thus, with neither depreciation method, the asset can be transferred at its book price. With the example numbers the difference between book and market price turns out to be practically identical.[19]

IV. Conclusion

The article shows that there are alternatives to the common method with which Neo-Ricardians and Ricardo himself treat fixed capital. While they are usually dismissed as obviously inconsistent and violating the profit rate equalization condition, I show that they can be made as consistent as the common method.

The key is to choose the profit charge on fixed capital in such a way that the internal rate of return on the investment in the fixed asset is equal to the normal rate of profit. The method that is designed to be annually cost consistent is thus shown to be also lifetime profitability consistent.

Consistent linear depreciation is a mirror image of the common method which is annual profit rate consistent, but violating the principle of annual cost consistency. Both techniques use financial mathematical formulas. When capital is modeled as “physical”, the proposed alternative method produces the identical price of production as the common one. When capital is modeled as “monetary”, where its value transfers from a machine into an accumulated depreciation fund, equilibrium prices differ from that of the common method if the interest rate is smaller than the normal profit rate. Thus, picking one correct method over another is economically relevant.

Monetary capital, where depreciation charges accumulate on a depreciation account is analyzed in greater detail. Different formulas must be used to compute amortization markups and book values of capital. They depend on the assumptions made about the credit system and the ability of the accumulation fund to obtain an interest rate.

[19] Of course, other problems like finding the cost-minimizing system of production occur when a system is interlocked. The prices of production used here may thus be different. A detailed treatment of these complications is, however, beyond the scope of this article. The essential argument of this paragraph holds - that the problem of differing book and market value occurs for all depreciation methods when a system is interlocked.

An interesting result is that the method proposed by the original Classics like Torrens and Marx is annual profit rate and cost consistent once capital is modeled in a monetary way and a simple credit system is assumed (no interest on the depreciation fund). This finding and the mathematical examples found in the writings of these two authors makes the interpretation possible that: 1. the original Classical economists actually did view capital in precisely this monetary way and 2. that they correctly applied this method to a stage of capitalism in which the credit system is not very developed.

REFERENCES

Bortkiewicz, Ladislaus Von. 1907. *Value and Price in the Marxian System.* London, U.K.:MacMillan and Company Ltd.

Deprez, Johan. 1990. "Vertical Integration and the Problem of Fixed Capital." *Journal of Post Keynesian Economics*, 13(1): 47–64.

Duménil, Gérard, and Dominique Lévy. 2000. "The conservation of value: a rejoinder to Alan Freeman." *Review of Radical Political Economics*, 32(1): 119–146.

Freeman, Alan. 2004. "Value from Nowhere: a response to Dumenil and Levy." Working Paper.

Gehrke, Christian. 2011. "The Joint Production Method in the Treatment of Fixed Capital: A Comment on Moseley." *Review of Political Economy*, 23(2): 299–306.

Keil, Jan. 2016. "Explaining the Concentration-Profitability Paradox." *Forthcoming.*

Kurz, Heinz, and Neri Salvadori. 1997. *Theory of Production: a Long-Period Analysis.* Cambridge University Press.

Kurz, Heinz, and Neri Salvadori. 2005. "Removing an 'Insuperable Obstacle' in the Way of an Objectivist Analysis: Sraffa's Attempts at Fixed Capital." *European Journal of the History of Economic Thought*, 12(3): 493–523.

Lager, Christian. 2006. "The Treatment of Fixed Capital in the Long Period." *Economic Systems Research*, 18(4): 411–426.

Malthus, T.R. 1825. "On the Measure of the Conditions Necessary to the Supply of Commodities." *Transactions of the Royal Society of Literature*, 1(1): 171–180.

Marx, Karl. 1990. *Capital Volume I.* New York, NY:Penguin Books.

Marx, Karl. 1992. *Capital Volume II.* New York, NY:Penguin Books.

Moseley, Fred. 2009. "Sraffa's Interpretation of Marx's Treatment of Fixed Capital." *Review of Political Economy*, 21(1): 85–100.

Moseley, Fred. 2011. "Reply to Gehrke." *Review of Political Economy*, 23(2): 307–315.

Panico, Carlo. 1988. "Interest and Profit in the Theories of Value and Distribution." PhD diss. University of Cambridge.

Pasinetti, Luigi. 1980. *Essays on the Theory of Joint Production.* New York, NY:Columbia University Press.

Perri, S. 1990. "The 'Transfer of Value' Approach to Fixed Capital." *Metroeconomica*, 41(3): 225–248.

Ricardo, David. 1951. *The Works and Correspondence of David Ricardo, Volume 1.* London, U.K.:Cambridge University Press.

Schefold, Bertram. 1989. *Mr Sraffa on Joint Production and Other Essays.* London, U.K.:Unwin Hyman Ltd.

Schefold, Bertram. 2011. "A contribution to Weber's theory of modern capitalism: amortization according to Sraffa as a rational substitution of missing markets." In *Keynes, Sraffa and the Criticism of Neoclassical Theory, Essays in honor of Heinz Kurz.* , ed. Neri Salvadori and Christian Gehrke, 179–197. New York, N.Y.:Routledge.

Shaikh, Anwar. 1980. "Marxian Competition versus Perfect Competition: further Comments on the so-called Choice of Technique." *Cambridge Journal of Economics*, 75–83.

Shaikh, Anwar. 2008. "Competition and Industrial Rates of Return." In *Issues in Finance and Industry, Essays in Honour of Ajit Singh.* , ed. John Eatwell and Philip Arestis, 167–194. New York, N.Y.:Palgrave Macmillan.

Shaikh, Anwar. 2015. *Capitalism: Competition, Conflict, and Crises.* New York, N.Y.:Oxford University Press.

Sraffa, Piero. 1960. *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory.* London, U.K.:Cambridge University Press.

Steedman, Ian. 1980. "A Note on the 'Choice of Technique' under Capitalism." *Cambridge Journal of Economics*, 61–64.

Tietze, Jürgen. 2009. *Einführung in die Finanzmathematik.* . 9 ed., Wiesbaden, Germany:Vieweg+Teubner.

Torrens, Robert. 1818. "Strictures on Mr. Ricardo's Doctrine Respecting Exchangeable Value." *Edinburgh Magazine*, 335–338.

Torrens, Robert. 1821. *An Essay on the Production of Wealth.* London:Longman, Hurst, Rees, Orme & Brown.

Vitaletti, Giuseppe. 2008. “The Optimal Lifetime of Capital Goods: a Restatement of Sraffa’s Analysis of Fixed Capital.” *Review of Political Economy*, 20(1): 127–145.

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Appendix – An Alternative Treatment of Monetary Capital In the analysis in this study, the interest on the depreciation account is considered a profit income category that is part of the computation of the annual profit rate. It is modeled such that it is immediately and freely available for free disposal or new investment. One could also treat it differently. One alternative is to use only the profit charge levied on the product, Π_t, as a profit income flow that is immediately and freely available. In this case, the interest income iF_t would then grow the accumulated depreciation fund and not be immediately and freely available for disposal and reinvestment.[20] The new accumulated depreciation fund at time t is then equal to

$$F_t = \sum_{n=1}^{t} \delta_n (1+i)^{t-n} \tag{A1}$$

and not as before, $F_t = \sum_{t=1}^{T} D_t$. δ represents new “depreciation” charges. These “depreciation” charges do not to sum up to the initial price of the fixed asset as before, where $K_1 = F_T = \sum_{t=1}^{T} D_t$. Instead, the requirement that the economic process can be repeated once the physical fixed asset is used-up has to be modified to

$$K_1 = F_T = \sum_{t=1}^{T} \delta_t (1+i)^{T-t} \tag{A2}$$

However, this produces a challenge. If $K_1 = F_T \neq \sum_{t=1}^{T} \delta_t$, but rather deter-

[20] I thank an anonymous referee for drawing my attention to this alternative method of treating monetary fixed capital.

mined by equation A2, then $K_1 = F_T < \sum_{t=1}^{T} \delta_t$ whenever $i > 0$. In words, the "depreciation" charges cannot be depreciating the physical fixed asset value to zero. Their sum has a numeric value smaller than the initial purchase price of the asset whenever the interest rate is greater than zero (there is no difference to the method outlined in this article if the interest rate is zero). Thus, δ_t now represents rather some type of "*corrective amount*" which has the function to form up the resources to buy again K_1 at the end of the asset lifetime.

This has an important implication: the computation of the values of the total capital stock, κ_t, the physical asset invested, K_t, and the accumulated depreciation fund, F_t is not straight forward anymore. If not the sum of all "corrective quantities" that replaced the former proper depreciation charges allow to derive the remaining value of physical capital ($K_1 - \sum_{t=1}^{T} \delta_t$), then what are the actual depreciation charges and what the prices of capital at different vintages? One could argue that proper depreciation D_t might still be treated linearly as in the annual cost consistent methods ($D_t = \frac{K_1}{T}$).

The correct corrective amount can be found using the accumulated savings account or future value of an annuity formula

$$K_1 = \delta \frac{(1+i)^T - 1}{i} \tag{A3}$$

Only the profit charges and the final release of the depreciation fund are available for free disposal or reinvestment. Thus, the correct M can be found from rearranging the coupon bond pricing formula with the internal rate of return $\tilde{r} = r^*$

$$K_1 = \Pi \frac{(1+r^*)^T - 1}{r^*(1+r^*)^T} + \frac{K_1}{(1+r^*)^T} \tag{A4}$$

to

$$\Pi = r^* K_1 \tag{A5}$$

Using the actual numbers from the numeric example, prices of production are identical to the ones obtained in the monetary method elaborated in the main body of the article. This is intuitive, since before, interest income was adding to annual profit quantities, while now it grows the depreciation fund. This in turn implies that the “corrective quantities”, δ, are necessarily smaller than what was previously called depreciation, D, if the interest rate is greater than zero. The net effect is exactly offsetting. Both views are equivalent with respect to resulting prices.

However, if the alternative method described here is followed, then the problem is that there are *two similar charges* which split up the functions depreciation previously had: a) depleting the fixed asset value to zero and b) rebuilding the financial means to repurchase it to K_1. This adds complexity for obtaining the same result. A complexity that was not present before. It also represent a non-standard way of dividing depreciation functions up into two separate charges or value changes.

The other part of the problem is the determination of the financial capital stock (formerly known as accumulated depreciation fund) in time t and of the total capital quantity, κ_t. Three answers are possible

1) the financial capital is the F_t implied by equation A1 (resulting in a total capital stock of $F_t + (K_1 - t * \frac{K_1}{T}))$;

2) there is no depreciation fund/financial capital (implying total capital be to only the physical fixed capital value remaining $(K_1 - t * \frac{K_1}{T}))$;

3) the financial capital is different from F_t, just like the value of physical capital is not $= K_1 - \sum_{t=1}^{T} \delta_t$. It could be that the depreciation fund value is then $= t * \frac{K_1}{T}$. This would imply that total capital would be a constant $= t * \frac{K_1}{T} + (K_1 - t * \frac{K_1}{T}) = K_1$.

None of these solutions are perfectly satisfying. The first and probably most intuitive answer implies a paradox outcome, where total capital is not just constant anymore. It even moves counterintuitively (for all the different possible positive interest rates) such that κ_t first falls from K_1 and then rises again back to K_1. If the second answer is the case, then the F_t becomes conceptually unclear. It does exist on the balance sheet of the company and does in fact represent the amount of money the capitalist saves from some type of depreciation charge to replace the physical fixed capital value. In the case of the third answer, it becomes even more challenging, since there are essentially two financial capital quantities at every point of time where only one represents the "true" depreciation fund that can be found on the bank account and the other one has no actual monetary substance. In any case it is not perfectly clear how the total capital stock. This makes creates uncertainty about the computation of the annual profit rates and about the application of an annual profit rate consistent solution.

In sum, the analysis seems to become more complicated due to multiple depreciation charges and uncertainty about vintage capital price and profit rate computations. It deviates from standard representations without effecting equilibrium prices and offering any real benefit. Comparatively, the method described in the main section of this article appears to be somewhat more transparent.