

**REFLECTION ON THE EFFECTS OF CONCRETE
MATHEMATICS MANIPULATIVES ON STUDENT
ENGAGEMENT AND PROBLEM SOLVING IN THREE
SECONDARY SCHOOLS IN TRINIDAD AND TOBAGO**

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This paper reports and reflects on three studies that explored changes in students' engagement in learning mathematics, and their ability to solve mathematics problems in three secondary schools in Trinidad and Tobago. Concrete manipulatives were integrated into mathematics instruction over 4 weeks in Trigonometry at two sites, and Set Theory at one site. The individual studies employed multi-method quasi-experimental, single-group action research designs. Student and teacher journals, observation checklists and notes were analysed using thematic analysis to identify themes related to student engagement. T-tests were computed to determine whether students perceived that engagement changed over their exposure to the intervention. Students' ability to solve mathematics problems was investigated via t-tests to determine whether their pre- and post-intervention achievement scores differed significantly, as well as qualitative analysis of their solutions to mathematics problems in the respective units. There was observable evidence of improvement in engagement, in learning, and problem-solving ability at the three sites. The findings across research sites suggested that students responded favourably to the integration of manipulatives into instruction. Reflections on

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these findings suggest that though they are specific to the three schools, they are consistent with research outcomes in research literature and support the integration of manipulatives into mathematics instruction to improve student engagement in learning and mathematics problem solving.

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Introduction

Globally, concerns have been raised about falling student achievement in mathematics, motivation to study mathematics, and interest in pursuing mathematics-related careers (Stokke, 2015; Wolfram, 2014). Similar reports are evident in Caribbean territories like Jamaica (Budoo, 2017) and Trinidad and Tobago (Kalloo & Mohan, 2015). More than half (52.3%) of students from Trinidad and Tobago who comprised the sample for the Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) 2015, were assessed as performing below the minimum acceptable performance in mathematics (Level 2). Further, the Caribbean Secondary Education Certificate Mathematics (CSEC) examination results from 2016 - 2018 reveal that Caribbean students are underperforming, with pass rates declining from 34% in 2016 to 24% in 2018 (Caribbean Examinations Council (CXC), 2018). Evidently, despite contextual differences among educational systems around the world, managing declining engagement and interest in learning mathematics and mathematics problem solving are problematic. One contributing factor to this phenomenon is the teachers' pedagogical decisions in the mathematics classroom.

Theorists like Piaget (1952) and Vygotsky (1978) suggested that children learn abstract mathematical concepts and relationships best when they have experiences that allow them to practice with concrete materials (manipulatives) rather than through lengthy and detailed explanations and demonstrations. This multi-representational approach to teaching mathematics appeals to different learning styles and provides opportunities for learning abstract concepts through sensory experiences (Ojose, 2008). It also facilitates the transition between concrete and abstract thinking to attain a deeper level of conceptual understanding. Therefore, it is no surprise that so many studies recommend the use of manipulatives in mathematics instruction (for example, Boggan, Harper & Whitmire, 2010; Konaş, 2016).

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Although manipulatives create enthusiasm for learning (Boggan et al., 2010), engage students in learning and can develop positive attitudes towards mathematics and improve academic outcomes (Holmes, 2013; Ojose & Sexton, 2009), they are not magical fix-all teaching resources (Ball, 1992). Unfortunately, the paucity of published empirical research in Trinidad and Tobago and the wider Caribbean about the use of manipulatives in the mathematics classroom does not lend support to either side of the argument. There is a gap in the research literature on small nation states, such as Trinidad and Tobago, that, like other countries, are struggling to capture and retain student interest in learning mathematics, and improve their academic achievement in mathematics.

Background

The authors of this paper are secondary school mathematics teachers and their university lecturers in the teaching of mathematics. The teachers were enrolled in a postgraduate in-service Diploma in Education programme at a university in Trinidad and Tobago that focused on developing their pedagogical skills. During this programme, the teachers undertook an action research project involving concrete manipulatives in their respective schools, under the supervision of the co-authoring university lecturers. The lecturers, having examined in-service teachers over the last 10 years, recognised that students benefit greatly in their engagement in learning and their ability to solve mathematics problems when concrete manipulatives were integrated into mathematics instruction; however, these gains have never been documented formally. Therefore, in 2017, the lecturers, recognising that these teachers had implemented similar interventions (concrete manipulatives), decided to formally document the observed outcomes of integrating concrete manipulatives into mathematics instruction, taking into consideration contextual and cultural differences among the schools, as well as nuances in the intervention itself, and methods across the sites. The authors documented evidence about outcomes regarding student engagement and

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problem solving at each site and reflected on the similarities and differences across school contexts. This paper provides an important point of reference for other teachers at secondary schools who might be interested in adopting this strategy.

Therefore, the purpose of this paper is two-fold. Firstly, the paper reports on how students responded to the integration of concrete manipulatives into mathematics instruction at three schools in Trinidad and Tobago, with respect to their engagement in learning mathematics and their mathematical problem solving. Secondly, the paper provides the authors' reflections after comparing and contrasting the findings and experiences from each site. This paper responds to the already established need for empirical research into instructional strategies to improve students' mathematics outcomes, particularly in educational contexts in which there is an apparent dearth of such research.

The studies focused on a unit on Trigonometry in Form 4/Grade 9 and Form 5/Grade 10 at two of the selected schools, and a unit on Set Theory in Form 1/Grade 6 at the third school. The studies at the three schools were guided by the following research questions:

1. How did the use of concrete manipulatives influence student engagement in learning mathematics at the selected schools?
2. How did the use of concrete manipulatives influence students' mathematical problem solving related to the units of instruction at the selected schools?

The Study Context

In Trinidad and Tobago, schools vary in several ways including context and culture; yet, regardless of the type of school, mathematics teachers may encounter similar challenges with low student engagement, and challenges with mathematical problem solving. The study was conducted at two government schools and one denominational school. De Lisle, Keller, Jules, and Smith (2009) described government schools as public schools owned, managed and funded by the government of Trinidad and Tobago,

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and denominational schools as schools owned and managed by a faith-based organisation, but partly funded by the government. The differences between these two types of schools extend beyond their management and funding, to include other factors, such as support from other stakeholders, including parents, past-students associations and non-governmental organisations. Additionally, entry into secondary school is based on students' scores on a highly competitive public, high-stakes placement examination, the Secondary Entrance Assessment (SEA), which is moderated by the government. The top-scoring students are placed at the most sought-after denominational and government schools that are generally perceived as top academic schools. De Lisle et al. (2009) suggested that this perception might not be accurate because formal information about the academic and non-academic attributes of these schools may not be available to parents and students. A brief description of each school involved in this study follows. The information presented was based on the teacher-researchers' personal knowledge about and experiences at the schools, during their tenure there.

School A. This seven-year denominational secondary school prepared students for the CSEC examinations at the end of Form 5/Grade 10, and Caribbean Advanced Proficiency Examinations (CAPE) at the end of Form 6 (equivalent to Grade 12). This school had a vibrant alumni group that actively organised fundraisers to support the school. Many students were from middle- and high-income families in which parents were typically highly-educated working professionals. The school's philosophy of holistic development propelled its investment in traditional academic subjects, sports and the performing arts. Students entering the school in Form 1/Grade 6 typically score in the top 20th percentile in the SEA and they often achieve excellent results in the top 10th percentile in CSEC and CAPE. Teachers at the school reported that they rely primarily on traditional instruction because of the heavy focus by the schools' stakeholders on these terminal examinations. According to the teachers, students view mathematics as boring and unrelated to their career goals and they demonstrate low levels of engagement in learning. Further, during the school year, students

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exert little effort on completing coursework and developing their problem solving skills, preferring instead to focus their energy on practicing to solve typical examination problems in preparation for CSEC.

School B. This seven-year co-educational government school had a student population of approximately 1,000. Students at the school excelled in sports and other non-academic activities, but performed poorly in traditional academic subjects, the least of which was mathematics, according to teachers at the school. Mathematics teachers expressed the view that students' poor attitudes towards mathematics were based on them perceiving it as abstract with little real-life applications. Consequently, student engagement in learning mathematics and their ability to solve even routine mathematics problems were low. On average, 30% of students obtained passing grades at CSEC Mathematics in any given year. Teachers reported that many students were from low-income households in which parents or guardians were themselves not proficient in mathematics. Suan (2014) associated parents' lack of proficiency in mathematics with students' ill preparedness to learn Mathematics.

School C. This five-year co-educational government school had a student population of approximately 700. Students entered the school with SEA mathematics scores averaging 40%, which teachers perceived as evidence that they lacked a strong mathematics foundation. Teachers indicated that students did not appear to value mathematics as important for their future careers, and seldom engaged in learning activities during instruction. Their ability to solve mathematics problems was weak, as demonstrated in their poor achievement scores during the school year. The school was heavily focused on non-traditional subjects, like technical-vocational, performing arts subjects and sports. However, teachers reportedly relied heavily on traditional instruction because they found the diversity of student learning styles and abilities challenging to manage. Many students were from low-income homes and some were suspected to have learning disabilities; but had not been formally diagnosed. Teachers reported that some

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students dropped out of school to pursue certification at trade schools.

Literature Review

Concrete vs Virtual Mathematics Manipulatives

Manipulatives are tools that are designed to stimulate children's mathematical development via their visual and kinaesthetic senses experienced through play and exploration (Uttal, 2003). They can be any objects in students' immediate learning environment. Larkin (2016) differentiated between concrete manipulatives (like tangrams, blocks and geo-strips) and virtual manipulatives (for example, Geogebra and Sketchpad). Concrete manipulatives are physical objects, like pictures, drawings, and 3-dimensional models that represent "a concept onto which the relationship for that concept can be imposed ... to illustrate and discover mathematical concepts, whether made specifically for mathematics (for example, connecting cubes) or for other purposes (for example, buttons)" (Van de Walle, Karp, & Bay Williams, 2013, p. 24). They facilitate "conscious and unconscious mathematical thinking" (Marshall & Swan, 2005, p. 14). Virtual manipulatives are digital representations of concrete objects that can be manipulated on a computer using a mouse (Moyer, Bolyard, & Spikell, 2002). The pervasive influence of technology has created a market for virtual manipulatives, which allow teachers to present students with pictorial, verbal and symbolic problems for exploration (Johnson, Campet, Gaber, & Zuidema, 2012).

Research has not conclusively determined whether concrete or virtual manipulatives are better for learning. Both types can enhance the learning environment and students' thinking (Siew, Chang, & Abdullah, 2013); develop students' conceptual understanding (Laskiet, Jordan, Daoust, & Murray, 2015) and spatial ability (Karakuş & Peker, 2015); and strengthen their ability to use multiple representations of mathematical ideas (Hakki, 2016; Karakuş & Peker, 2015; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009); and students' interest, involvement, retention and

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mathematics achievement (Hakki, 2016; Larkin, 2016). Therefore, teachers should select manipulatives that assist with concept and skills development, and are appropriate for the mathematics content to be taught.

In these studies, teachers selected concrete manipulatives because of the challenges of accessing technology to support the use of virtual manipulatives at the three schools. They adhered to Sarama and Clements' (2016) guidelines for using manipulatives effectively across contexts: using them to encourage exploration through meaningful and appropriate play; using only those that were appropriate for the context, being judicious with commercially-produced manipulatives that may not permit flexibility in their use; using them as introductory symbols to model problems and represent mathematical ideas, and then transitioning to drawing and symbolic visualisation.

Why Use Manipulatives in Mathematics

Theorists like Piaget (1952), Dienes (as cited by Sriraman & Lesh, 2007), Bruner (1986), Skemp (1987), Dewey (1938) and Vygotsky (1978), have advocated for the use of manipulatives in mathematics instruction. Piaget (1952), Bruner (1986) and Skemp (1987) opined that learners must first experience ideas at the concrete level before they can understand symbols and abstract concepts. Dienes, during his interview with Bharath Sriraman (Sriraman & Lesh, 2007), argued that manipulatives allow students to construct their own conceptual understanding of mathematics through active participation and collaborative group work, which supports Vygotsky's (1978) theories around teachers pushing students through their zone of proximal development by having them use manipulatives to explore and discuss mathematical ideas with others.

Dewey (1938) advocated for teachers to create learning experiences that facilitate practice that is embedded in the environment. Later, Bruner (1986) was critical of teachers who teach mathematics without linking it to students' daily experiences

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in the environment. He theorised that children progress through three stages of learning, from the concrete to the representational, and then the abstract phase. In the concrete phase, students would be first exposed to manipulatives and engage in explorations guided by the teacher. They represent their understanding of mathematical problems to create models that they use to solve problems. In the representational phase, students begin to become independent of manipulatives and can represent their understanding of mathematical problems mentally or with diagrams that they use to solve problems. In the abstract phase, students are able to think symbolically and no longer rely on manipulatives to represent problems, although they may still rely on mental images or diagrams to help them solve problems.

Concrete Manipulatives and Student Engagement in Learning

Engagement in learning is a complex and difficult construct to define and measure. However, employing pedagogical strategies that engage students, can have a positive impact on student learning in mathematics (Fielding-Wells & Makar, 2008). The research on engagement in learning identified three interrelated dimensions: affective, behavioural and cognitive (Fredricks, Blumenfield, & Paris, 2004). These dimensions guided the development of indicators of engagement utilised by the researchers in their studies. Affective engagement encompasses beliefs and attitudes and is exemplified through students demonstrating interest in lessons. This dimension was related to students willingly attending classes, being punctual, staying for the duration of the session and being prepared with the tools and equipment needed to perform mathematical tasks for this paper. Behavioural engagement focuses on proper conduct and this was observed in these studies by indicators, such as asking questions relevant to the lesson; listening and paying attention to the teacher and other students. Cognitive engagement targets attention to information, and using it to solve problems with resilience and flexibility, as well as a desire for challenge. Indicators of this dimension in these studies included, staying on task, completing class and homework assignments and using manipulatives with fidelity.

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The use of concrete manipulatives can create a learning environment that supports student engagement in learning mathematics. Shaw (2002) reported that the use of manipulatives engaged students in kinaesthetic learning experiences that allowed them to explore and understand mathematical relationships. Spear-Swerling (2006) noted that increased student engagement in learning and their understanding of difficult concepts were observed during learning with concrete or virtual manipulatives. Further, Cockett and Kilgour (2015) reported that concrete manipulatives helped with improving students' positive perceptions of the learning environment, decreased students' off-task behaviours, and prolonged active engagement in learning activities with concrete manipulatives. These studies provided sufficient support for the use of concrete manipulatives to engage students in learning mathematics, which aligns with the purpose of the studies reported in this paper.

Concrete Manipulatives and Mathematical Problem Solving

Beyond discussing mathematical ideas, research also supports the use of manipulatives for developing students' mathematics problem solving. Lester and Kehle (2003) described problem solving as an activity that engages students in cognitive processes that include retrieving and utilising appropriate pre-knowledge, experiences, representations (including manipulatives), and patterns to explore relationships and conjectures that aid with problem solving. Kelly (2006) describes the problem-solving process as an iterative process involving understanding a problem, devising a solution, executing and revisiting the problem to verify the solution and reflecting on the process. In the studies being reviewed in this paper, the teacher-researchers introduced students to the problem-solving process through the use of concrete manipulatives. This allowed them to develop their conceptual understanding and derive mathematical relationships using concrete objects, which prepared them to apply their knowledge to solving mathematical problems. Kelly (2006) drew on Reusser's (2000)

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argument that children construct their mathematical knowledge and skills by actively engaging with others and objects in their natural environment. Reusser (2000) suggested that we typically solve daily problems that involve some tangible object, and teachers can mimic this approach with students in the classroom through the use of concrete manipulatives.

Van de Walle, Karp, and Bay William (2009) and Moyer (2001) advocated for developing students' intuitive understanding of mathematical relationships before encouraging them to learn the rules for their use. In other words, although concrete manipulatives themselves do not directly illustrate mathematical relationships, students develop a deeper understanding of the mathematics they are learning through explorations of mathematical relationships that manipulatives can help model. Consequently, they become more confident in their ability to apply mathematical ideas to solve even challenging problems (Cockett & Kilgour, 2015). Therefore, manipulatives could be used to engage students in thinking and making connections to real-life scenarios, which move them beyond the point of automaticity. In fact, Martin (2009) postulated that the use of the same manipulatives repeatedly to solve problems could deepen understanding of basic concepts. It is no surprise, then, that in 2013, the National Council of Supervisors of Mathematics advised educators to integrate the use of manipulatives into mathematics instruction, at all levels, to develop students' mathematics proficiency.

Caveats to Mathematics Manipulatives

Unfortunately, using manipulatives does not guarantee students meaningful learning experiences (Uttal et al., 2009) and may not improve achievement (Carbonneau, Marley, & Selig, 2013). Thus, introducing manipulatives to students with too little or inappropriate instruction and guidance from the teacher, can produce off-task student behaviour, while too much instruction can suffocate students' creativity and meaning-making from the experience (Carbonneau et al., 2013). Further, some students find manipulatives distracting, particularly when they closely resemble everyday items, as McNeil, Uttal, Jarvin, & Sternberg (2009)

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learned in their study that used realistic play money. The caution here is that superficial characteristics of manipulatives can distract students from problem solving (Carbonneau et al., 2013). Hence, teachers must judiciously select manipulatives that focus students' attention on the mathematical concept it represents and nothing else. Teachers must also ensure that manipulatives are used for enough time for students to connect them to mathematical concepts (Laski et al., 2015). This can be achieved by re-using the same manipulative to solve problems and deepen understanding of basic concepts (Martin, 2009). While commercially available manipulatives can be costly, teachers can select cheaper versions or make them from readily available materials; either way, from Hakki's (2016) perspective, the potential benefits of their use outweigh their costs. One final caution from Larkin (2016) is that manipulatives become less effective with students over the age of 12 years, but this should not deter any teacher from incorporating them into instruction with any group of students.

Methodology

Design

This paper reports on three action research studies that addressed issues three teachers faced in their classrooms with student engagement in learning mathematics and their mathematics problem-solving at three schools in Trinidad and Tobago. The merit of the action research design, is that it provides insight into how students learn (Sagor, 2000) and, according to Glanz (2014), affords educational practitioners systematic and cyclic enquiry into their practices, assessment procedures and interactions with students with the goal of solving a specific problem, which in this case involved student engagement and mathematical problem solving. The value of this paper, then, is that it reports on understandings gleaned through scholarly research into teacher-researchers' classroom practices in varying school contexts.

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The studies involved reconnaissance and implementation as outlined by Stringer (2013), at each school. Using multiple data collection methods facilitated data triangulation and complementarity (Greene, Caracelli, & Graham, 1989) within each of the three sites, and aided with understanding the outcomes of the interventions, and extending the breadth and range of inquiry (De Lisle, 2011). In the end, the authors reflected on the outcomes across the three sites, cognisant of the contextual differences among the schools, in effect, learning from their own experiences (Leitch & Day, 2000) about how the use of manipulatives can influence student engagement in learning and problem solving in mathematics.

Sample

The studies used purposive sampling to select one class from each of the three research sites, an approach recommended by Yin (2014) to select sites and samples appropriate for research goals. Further, criterion-based sampling was used to establish criteria for selecting samples at each research site, an approach recommended by Goetz and LeCompte (1984). Primary criteria for inclusion in this study were evidence of the phenomenon of low student engagement in learning and difficulty with problem solving in mathematics, as well as negotiated access to each site (Pettigrew, 1990). The samples at the three schools are described next.

School A. This sample was a Form 4/Grade 9 class (ages 14 – 17 years) comprising 30 boys. The class comprised student-athletes who represented the school in various sports and others with key responsibilities in extracurricular clubs. Generally, students demonstrated low engagement in learning, as evidenced in their high energy levels and mildly disruptive behaviour during teaching: being easily distracted, tardiness to class or truancy, non-completion of seatwork and homework, and poorly developed problem-solving skills. Their mathematics grades over the previous term averaged 50%. In particular, these students experienced difficulties in the study of topics in geometry. For example, they found it challenging to relate the relationships between angles and sides in shapes, to see

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its value in the real world, and its relevance to their future career goals.

School B. This sample was a Form 5/Grade 10 class (ages 15 – 17 years) comprising 12 girls and 9 boys. It was considered a vocational class because the students all pursued vocational subjects like cosmetology, tailoring, and food and nutrition. Generally, students demonstrated low engagement in learning, as evidenced by private unrelated discussions among groups during teaching. Some students also placed their heads on the desk, often falling asleep. Students were unresponsive to the teacher's attempts to engage them in learning. They experienced difficulties with problem solving, preferring to wait for the teacher to provide solutions. Their average mathematics grades over the previous term ranged from 34% to 46%. These students often had difficulty visualising and modelling real-life situations and were unable to diagrammatically represent and solve word problems related to trigonometry accurately while in Form 4/Grade 9.

School C. This sample was a Form 1/Grade 6 class (ages 11 – 13 years) comprising 8 girls and 16 boys. This mixed-ability class was selected because students demonstrated low engagement in learning, as evidenced by their truancy or tardiness to class, disruptive behaviour during teaching, unresponsiveness to the teacher's attempts to engage the students in learning, lack of school supplies, and non-completion of seatwork and homework. Their average mathematics grades over the previous term ranged from 15% to 96%. These students also struggled with problem solving, but were very excited by any opportunity that was tactile in nature.

Intervention Procedure

The interventions at the three schools were conducted from January to March 2018. Prior to the intervention, under the guidance of their research supervisors, the teacher-researchers obtained permission from their school's administration and informed consent from parents or guardians of participating students. They created unit plans and lesson plans with appropriate manipulative activities

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for each lesson prior to the intervention at each school. At each research site, prior to implementing the intervention, the teachers exposed students to the particular manipulatives, the journal writing process, and administered questionnaires that captured students' perceived engagement in learning mathematics. During the intervention across the three schools, the last five minutes of each lesson was dedicated to students' journaling about the lesson. Upon completion of the intervention, students completed the same questionnaire again as well as a summative assessment on the specific unit of work. The specifics of the interventions at each school follow, and include any additional data collection conducted.

School A. The study focused on a unit of trigonometry that introduced trigonometric rules and theorems specific to the right triangle (Pythagoras' theorem, sine, cosine and tangent trigonometric ratios), and problem solving involving real-world examples including aeronautics, medicine and sport. Geostrips were the manipulatives selected to aid with learning and concept retention (Marzano et al., 2001). Geostrips are short flexible sticks that snap together and can be manipulated to form various angles and polygons. The teacher made the geostrips by punching holes at regular intervals in 6" and 8" wooden craft sticks that could be connected with brass fasteners and miniature clothespins to create triangles (see Figure 1). The distance between pairs of holes represented 1 unit length.



Figure 1 Sample of geostrips used at School A

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The decision to use geostrips at School A, a single-sex boys' school, was supported by the findings of Carbonneau et al. (2013) who reported that greater benefits could be gained by the use of more non-descript manipulatives, particularly with male students, as observed by Siew et al. (2013) and Olkun (2003). Research provides favourable outcomes of manipulatives in improving geometric thinking (for example, Karakuş & Peker, 2015; Larkin, 2016; Siew et al., 2013), particularly when they are concrete manipulatives (Olkun, 2003) and related to the study of trigonometry (Brijlall & Niranjana, 2015).

Students used geostrips over the equivalent of ten 35-minute lessons to represent scenarios that could be modelled using right-angled triangles, to facilitate manipulation and problem solving (Mullan, 2002). They created different-sized triangles and used rulers and protractors to measure interior angles and lengths of sides to explore trigonometric relationships, before transitioning to abstract representations of these relationships when students would work without the geostrips. The teacher-researcher used a behaviour checklist to record student engagement during each lesson, and made jottings on the checklist to note any observed behaviours that were not captured in the checklist.

School B. The study focused on a unit of work in trigonometry that introduced the sine, cosine and tangent ratios and focused on problem solving using these ratios. The manipulatives used were action figures, dowel rods, Styrofoam blocks and angLegs. AngLegs are coloured plastic line segments of varying lengths that are connected to create lines, angles and polygons by snapping their ends together. Tabor (2013) blogged about how this resource brings geometrical concepts to life for middle and high school students, and helps them make sense of geometry vocabulary in context. Dowel rods are coloured plastic rods of varying lengths that can be used to create geometric shapes. Figure 2 illustrates these resources. Brijlall and Niranjana (2015) reported that Grade 10 students in a South African school demonstrated improved abilities to visualise and understand word problems when miniature figurines

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were used to represent real-life situations. These manipulatives would showcase students' visual and kinaesthetic ability to create a mathematical model. These manipulatives were perceptually rich, meaning that they resembled real-life objects. Therefore, they would provide greater opportunities to closely model real-life situations and these models can aid in improving students' abilities to visualise space and shape (Brijlall & Niranjana, 2015).



Figure 2 Sample of Manipulatives used at School

Teaching occurred over the equivalent of nine 35-minute lessons. Students used dowel rods, action figures and Styrofoam blocks to model word problems. They then used angLegs to create right-angled triangles derived from a given word problem. After students identified the reference angle on the angLegs, they used a dry erase marker to label angLegs with “hypotenuse”, “adjacent” and “opposite” with respect to the said angle. Following this, they traced their model into their notebooks and replicated the labelled sides and angles. Then they used the information to identify the appropriate trigonometric ratios to solve the word problems. The teacher-researcher wrote her observations about student engagement in the tasks in her journal during each lesson.

School C. The study focused on set theory that introduced students to its concepts, applications and problem solving. Learning set theory is sometimes challenging for students. Manipulatives provide students with hands-on opportunities to observe concepts and relationships unfold. The manipulatives included familiar objects, such as string, plastic bottle caps and bottles, Styrofoam cups, foam cubes, balloons, flash cards and craft sticks. These familiar objects were selected due to their lack of perceptual-richness, which can distract students from the use of manipulatives

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for the designated tasks, and from recognising what the manipulatives are representing (Petersen & McNeil, 2013).

Teaching occurred over the equivalent of eight 35-minute lessons. In lessons in which students formed sets: they created sets by grouping the different manipulatives according to different characteristics like colour, shape, size. Modelling clay was used to enclose the sets. In the lesson on the union of two sets, number strings were created using foam cubes, and each string had one blank cube (see Figure 3). Cubes were rotated to the required numbers to form various sets of numbers, which were tied together to represent the union of two sets. Where a number was repeated, the blank cube was turned upright.



Figure 3 Manipulatives Used at School C

The teacher-researcher used a behaviour checklist to record student engagement during each lesson and made jottings in her teacher journal about any observed behaviours that were not captured in the checklist.

Data Collection and Analysis

Data were collected across the three schools using a variety of methods (see summary in Table 1). A brief discussion of each method and related analysis technique follows.

Table 1 Summary of Methods of Data Collection and Analysis

Research Question	School	Data Collection	Data Analysis
1. How did the use of concrete manipulatives influence student engagement in learning mathematics at the selected schools?	A	Student engagement survey Student Journal Observation Checklist	Descriptive statistics Paired sample t-test Thematic analysis
	B	Student engagement survey Student journal Teacher Journal	
	C	Student engagement survey Observation checklist Student journal Teacher journal	
2. How did the use of concrete manipulatives influence students' ability to solve problems related to the units of instruction at the selected schools?	A	Base scores and Post-test	Descriptive statistics Paired sample t-test
	B	Pre-test and Post-test	
	C	Base scores and Post-test	

Research Question 1: Student Engagement in Learning Student Surveys

Students were surveyed using teacher-constructed self-report questionnaires to determine their perceived engagement in learning mathematics at the start and end of the intervention. Questionnaires allowed respondents to carefully consider their responses without interference (Miller, 2002). The items on the questionnaires were designed to align with the literature on student engagement in learning mathematics, and to reflect the contextual differences and nuances that distinguished poor engagement at each particular research site. Each questionnaire addressed the issue under investigation: student perceived engagement in learning mathematics. Items were 5-point Likert items with predetermined responses that were weighted from *strongly disagree* (weighted as 1) to *strongly agree* (weighted as 5) for positively phrased statements, and the reverse weighting for negatively phrased statements. Students' responses were weighted and tallied for analysis. Descriptive statistics were computed for each dataset (frequencies, means, standard deviations, ranges), followed by paired-samples t-tests on each dataset to determine whether there were statistically significant differences in students' responses. A description of each questionnaire follows.

School A. This questionnaire comprised 10 items that addressed students' perceptions about mathematics their engagement in learning mathematics, including: "I like learning about math.", "I pay attention in math class most of the time.", and "I complete exercises, including homework, given for math class most times." The total possible score on the questionnaire ranged from 10 to 50.

School B. This questionnaire comprised 15 items about students' perception of their engagement in learning mathematics, including: "Mathematics is easy to understand.", "I often daydream during mathematics class." and "I like to answer questions during mathematics class." The total possible score on the questionnaire ranged from 15 to 75.

School C. This questionnaire comprised 11 items about

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students' perception of their engagement in learning mathematics, including: "I make excuses to leave class.", "I make an effort in class to answer questions." and "I never have the material I need to participate in class." The total possible score on the questionnaire ranged from 11 to 55.

Student journals. Reflective journals allow students to articulate aspects of a lesson that impact them most (Phelps, 2005), and provide insight for the teacher-researcher about their thoughts and feelings about the intervention strategy overall. Even when students' experiences are indirectly related to the lesson, for example peer-to-peer interactions, they may stimulate learning by acknowledging and validating these experiences through journaling. Prompts were provided to focus students' attention on their engagement in learning during each lesson: what they liked, what they would change about the lesson, what they wanted to spend more/less time on and any additional thoughts. Data were analysed using thematic analysis that focused on "systematically identifying, organising and offering insights into patterns of meaning (themes) across a dataset ... to make sense of collective or shared meanings and experiences ... relevant to answering a particular research question" (Braun, Clarke, Hayfield.& Terry, 2019, p.57). In this study, themes were related to indicators of student perceived engagement in learning mathematics.

Teacher Journals. Journals assisted teachers in documenting how the lesson unfolded, and any noteworthy events; a critical process during action research beyond recalling events. Reflection in the classroom is a crucial part of the teaching process. The teacher becomes the researcher in context (Schön, 1983). Teachers at Schools B and C used a teacher's journal, but the teacher at School A preferred to make jottings on her student checklist during and after each lesson. Teachers wrote in their journals as soon as possible after each lesson. They documented their experiences and observations about students' responses to the use of concrete manipulatives, and their engagement in learning during each lesson. Thematic analysis was utilised to reveal themes related to student engagement.

Observational Checklist. Checklists guided teachers'

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observations of student engagement during the lessons. Morrison, Ponitz, and McClelland (2010) endorsed the use of checklists to track learners' progress. Teachers created a list of observable behavioural indicators of engagement, which they used to record the occurrence (or absence) of identified behaviours during each lesson. These behavioural indicators included student attendance, punctuality, requests to leave the classroom during the lesson, preparation for class (books, writing equipment and mathematical tools), completing homework, asking questions relevant to the lesson, listening and paying attention to the teacher and other students, appropriate use of manipulatives, and staying on task during the lesson. Checklists were used in Schools A and C, but not at School B. The teacher at School B preferred to record her observations as jottings that she elaborated on in her teacher journal, to allow her to record as many classroom events as possible.

Research Question 2: Mathematical Problem Solving

To assess students' mathematical problem solving, they were administered a teacher-created summative assessment related to the units of work. The teacher-researchers analysed the quality of students' responses to items on the assessments to provide a qualitative interpretation of their mathematical problem solving. Subsequently, they each computed a paired samples t-test (2-tailed, .05 significance level) to determine whether there was a significant difference between students' baseline scores (Schools A and C) or pre-test (School B) scores at the start of the intervention, and their summative assessment (post-test) scores at the end of the intervention. Baseline scores were computed from students' coursework in the previous term. All scores were reported as percentages. A description of each assessment follows.

School A. The assessment comprised five story problems that assessed students' knowledge and application of Pythagoras' theorem and trigonometric ratios (sine, cosine and tangent) to model given scenarios using right-angled triangles and solve them. The first question required students to recall the three trigonometric

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ratios, and the second question required them to state Pythagoras' theorem. The next three items involved students' ability to select and apply Pythagoras' theorem; the three trigonometric ratios to calculate the length of unknown sides and sizes of unknown angles in isosceles and right-angled triangles; and a compound shape involving two triangles.

School B. The pre-test/post-test approach was utilised in School B because a unit of trigonometry was taught during the previous term using a primarily teacher-centred approach that did not focus on transitioning from concrete to abstract thinking, and meaningfully engaging students in learning. Identical tests were administered to students prior to and upon completion of the intervention. The assessment comprised two story-problems requiring students to diagrammatically model real-life situations using right-angled triangles, and one problem involving a compound shape comprising two triangles. The three items required students to select and apply appropriate trigonometric ratios to calculate the lengths of missing sides and the size of a named angle.

School C. The assessment comprised 10 multiple-choice questions and four short-response questions. Students were required to name sets using words and set notation, and describe the elements of a set and of subsets of a set. They were also required to interpret given pairs of sets and apply their knowledge of the union and intersection of two sets to list the elements of these new sets formed and represent these sets on Venn diagrams.

Ethical Considerations

The teacher-researchers adhered to a number of internationally recognised ethical protocols with respect to anonymity, privacy and confidentiality (Hammersley & Traianou, 2012) to avoid jeopardising the research process. Schools were assigned pseudonyms and participants were assigned numeric codes. All participants (schools' administrators, parents/guardians and students) were provided detailed information about the studies for decision-making regarding their participation, and the option to not participate or withdraw at any time without repercussion. Informed

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consent was obtained from students and their parents/guardians. Additionally, they were fully apprised about the nature of the study, the expected outcomes, and benefits to them. This approach has been demonstrated to increase participant rates (Blom-Hoffman et al., 2009).

Limitations

The action research design is not without limitations. There are two that stand out in the studies conducted. First, the use of non-random sampling and the relatively small sample of schools and students in the studies suggest that findings may be relevant only to the classes under investigation, and meaningful generalisations cannot be made (Ary, Jacobs, Sorensen, & Razavieh, 2010). However, the research team did not generalise findings, rather they compared the findings across research sites to gain a deeper understanding of participants' responses to the intervention in their particular contexts with respect to engagement in learning and mathematical problem solving. It is, in fact, possible that these findings could be comparable to sites that share some similarities to the schools in this study, and may be interpreted through those lenses, a phenomenon Morse (1999) refers to as situational comparability. Additionally, according to Cockett and Kilgour (2015), limiting the timeframe within which action research is conducted, can also introduce a limitation, which can be mitigated by cautiously reporting the research findings.

Findings

The findings are presented with respect to students' engagement in learning and their mathematics problem solving, for each of the three schools involved.

Students' Engagement in Learning Mathematics

School A. The highest possible score on the student engagement survey was 55. Analysis indicated that 25 of 30 students reported higher scores after the intervention; half of them scored

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between 40 and 50 post-intervention compared to only one quarter of them pre-intervention (see Figure 4). A paired sample t-test indicated a significant difference in students' perceived engagement, $t(29)=4.67$, $p < .05$, from pre-intervention ($M = 39.60$, $SD = 5.28$) to post-intervention ($M = 36.27$, $SD = 6.31$).

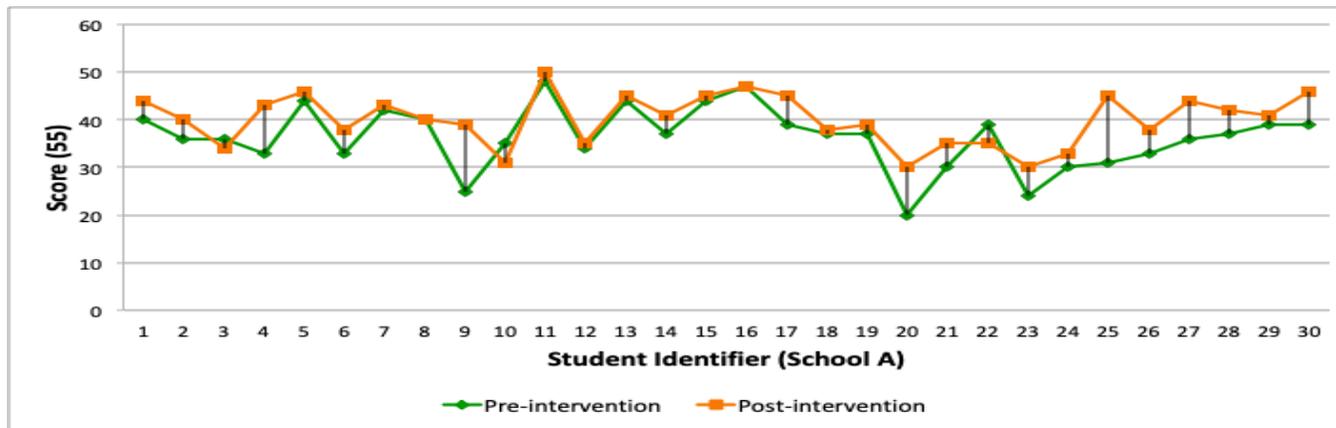


Figure 4 Students' Scores on Engagement Questionnaire

Thematic analysis of students' journal entries supported the positive change in students reported engagement in the questionnaire. Generally, most boys did not appreciate the journaling activity at the beginning of the intervention, and many of their journals reflected a lack of detail in responding to the provided prompts. At the end of the very first lesson, 10 students wrote "nothing" in response to some journal prompts, but by the fourth lesson, all students had provided some response to the prompts. By the sixth lesson, most responses to journal prompts were more detailed and referred specifically to the lesson content (see Table 2).

Table 2 Emerging Themes and Excerpts from Students' Journals

Enjoying hands-on activities	Connection to the world	Active engagement
My favourite part of today's lesson was crafting of a square, which helped in understanding Pythagoras theorem.	My favourite part of today's lesson was learning about airplanes and their sine ratio application.	If I could change any part of the lesson I would change nothing, I liked how interactive it was.
I liked creating triangles to trace in my book.	My favourite part of today's lesson was learning how doctors use trigs and how they use the cosine ratio to apply metal plates.	I participated by folding and cutting the paper to stick in my book to represent Pythagoras' theorem.
I enjoyed cutting the sheets and using the manipulative to do the exercise.	My favourite part of today's lesson was the PowerPoint on the uses of the ratio and using the manipulatives to create an arm with a needle in it	I participated by crafting triangles to aid in my discovery of sin, cosine, tangent of triangles.

Analysis of the teacher's observation checklist supported the increase in student engagement, and an overall improvement in classroom behaviours. Generally, student deviances were at their highest at the start of the action research, but dissipated as it progressed. For instance, the number of students that were either tardy, asked to leave the class or were unprepared for the first lesson were 4, 5 and 7, respectively. The numbers of students who were tardy and asked to leave the class in the fourth lesson were 0 and 2, respectively. By the last two lessons, the number of students asking questions relevant to the lesson, listening and paying attention to the teacher and other students, appropriately using the manipulatives, and staying on task during the lessons increased from about half of the class to all students. Therefore, it was concluded that students were more productive throughout the lessons and engaged in using the geostrips for learning (see Figure 5).



Figure 5 Student Manipulative Models Using Geostrips

School B. The highest possible score on the student engagement survey was 75. There was an overall increase in most students' scores on the self-report engagement questionnaire at the

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end of the intervention (see Figure 6). The paired samples t-test revealed a significant increase in mean scores on the engagement questionnaire, $t(21) = 5.248$, $p < .05$, from pre-intervention ($M=37.48$, $SD = 9.39$) to post-intervention ($M=48.43$, $SD = 13.83$).

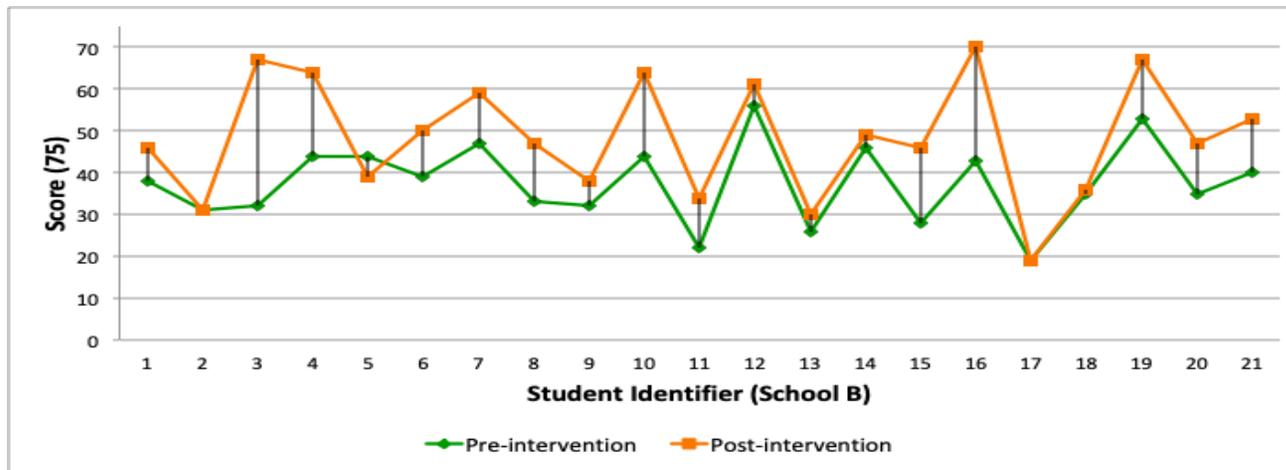


Figure 6 Students' Scores on Engagement Questionnaire

Students' journals supported the increase in engagement they reported in the questionnaires. Thematic analysis revealed three themes: student enjoyment; opportunities to practise; and opportunities for group work. Table 3 provides students' words to support these themes. These themes suggest that students enjoyed working with manipulatives and were more engaged during the sessions using manipulatives. They were, also, more inclined to complete assigned tasks using manipulatives. It must be noted that students' initial set of journal entries were not as elaborate as in Table 3, but the level of descriptive detail in their responses increased over time from the first to the last lesson. Table 3 provides responses from the latter half of the lessons, though they reflect what students wrote in their first set of journal entries.

Table 3 Emerging Themes and Excerpts from Students' Journals

Enjoyment	Opportunities for Practice	Opportunities for Group Work
I enjoyed using the models/ action figures to solve worded questions in trigonometry.	I wish I could have spent more time using the manipulatives	I enjoyed working with my group, it helped me to better understand the work when someone is there to help.
I enjoyed using the action figures, dowel rods and styrofoam blocks.	I wish I could spend more time finding the sides using the action figures, dowel rods and styrofoam blocks	I enjoyed explaining the questions to my group members and building the figure.
I thoroughly enjoyed figuring out how to arrive at the answer.	I wish I could have spent more time doing questions	I wish we could have spent more time interacting with our group members.
I really enjoyed the lesson, I just wish it could go on a little longer.	I wish that we spent more time with the GeoGebra on the board.	I enjoyed the games and the group activities.

Figure 7 illustrates how students used the manipulatives to model story problems.



Figure 7 Manipulative Models Using Dowel Rods, Styrofoam Blocks and angLegs.

The teacher's observations recorded in her journal supported the observable increase in student engagement in learning with manipulatives. Thematic analysis revealed three themes from the teacher's journal entries: student enjoyment; opportunities for practice; and opportunities for group work. Table 4 presents direct quotes from her journal.

Table 4 Emerging Themes and Excerpts from Teacher's Journals

Student Enjoyment	Opportunities for Practice	Opportunities for Group Work
Students enjoyed working with the virtual manipulatives today.	S002 completed the question and asked to try other questions.	I think working in groups benefited S020 today. Her group members were able to assist her in correctly using the angLegs.
Students were enthusiastic about keeping angLegs to practise.	Two out of the four groups were able to transition very quickly between modelling the problem and solving. They kept asking to solve more questions.	Students seem to like working together in explaining concepts to each other.

School C. The highest possible score on the student engagement survey was 55. There was an overall increase in most students' scores on the self-report engagement questionnaire at the end of the intervention (see Figure 8). The paired samples t-test revealed a significant increase in mean scores on the engagement questionnaire, $t(22) = 4.169$, $p < .05$, from pre-intervention ($M=42.87$, $SD = 7.59$) to post-intervention ($M=47.87$, $SD = 5.49$).

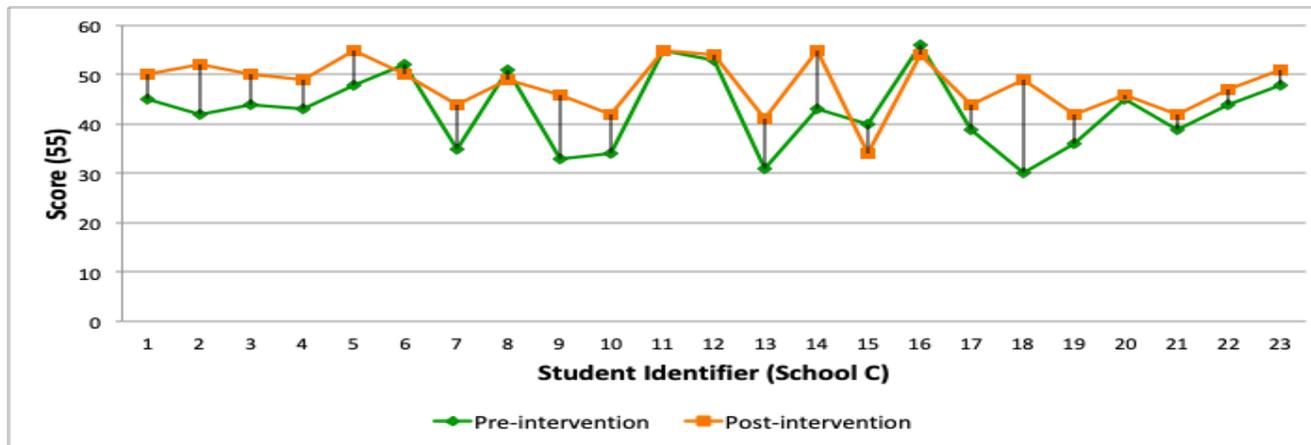


Figure 8 Comparison of Students' Pre- and Post-Intervention Engagement Questionnaire Scores

Students' journal entries supported an increase in their engagement in learning. It must be noted that these students struggled with writing, and their journal entries were not well elaborated in the beginning. However, over time students wrote more elaborate responses in their journal entries. The disparity in their responses is represented in Table 5. Thematic analysis revealed three themes: increased participation, student enjoyment; and increased enthusiasm. Table 5 provides students' words to support these themes. These themes suggest that students enjoyed working with manipulatives, participated more in the lesson and were more engaged during the sessions using the manipulative.

Table 5 Emerging Themes and Excerpts from Students' Journals

Increased participation	Student enjoyment	Increased enthusiasm
I raised my hand.	Class was fun. There was less writing and more fun.	I wish all miss classes are like this.
I answered more questions because miss made it fun by using different kinds of things.	My team won!	I like maths more.
I participated today by using cups to explain and answer all of our teams questions correctly.	We got to play with things today	I want to get to play with more things next class.

Thematic analysis of the teacher's journal revealed three themes: increased collaboration, excitement to learn, and increased enthusiasm. Table 6 provides the teacher's words to support these themes. These themes suggest that students were more engaged and enjoyed working collaboratively with manipulatives. They were more enthusiastic and excited to learn using the manipulative.

Table 6 Emerging Themes and Excerpts from the Teacher's Journals

Increased collaboration	Excitement to learn	Increased enthusiasm
Today students were working in groups. They were eager to complete the activity to beat their classmates in the other groups.	They would high-fives their friends when they were correct. -> Super excited and happy!	Kids were excited for the class to see what they were getting to use today. They were more active and energised.
Students were talking more amongst peers in order to figure out a question. It was good to hear small argumentative discussions among the students.	When they were asked to figure out something they were very involved and engaged in the discussion with their friends.	They were shouting their answers and rushing to write it on the board. Kids were more willing to come up with suggestions and methods.
Many of the students were adopting the role of a teacher in their groups in order to help those in difficulties.	Some shouted, "Miss, pass back. We not ready with our stuff yet!"	Students were fixing desks quickly to get the class started. Glad to see such enthusiasm.

Analysis of the teacher's observation checklist supported the increase in student engagement and an overall improvement in classroom behaviours, indicating more student engagement in the lessons over time. For example, over time students were increasingly punctual to class and prepared with books and pens, volunteered to respond to questions and work on the board, actively engaged in learning activities using manipulatives (see Figure 9) and class discussions and, by the end of the intervention, most of them completed homework tasks.

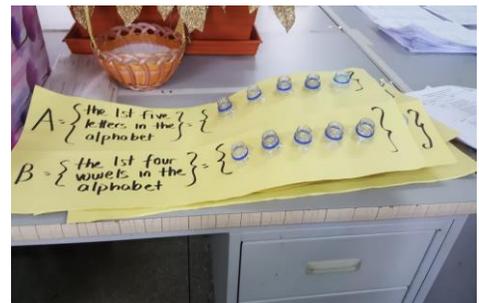
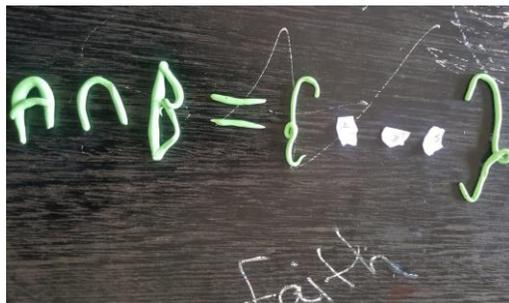


Figure 9 Samples of Manipulatives used to Teach Set Theory

Overall, these findings suggest that students responded favourably to the manipulatives during instruction. They were more engaged in learning activities during instruction, arrived to class promptly, were prepared with their books and other materials, participated in class discussions more frequently and willingly, and appeared more focused on learning.

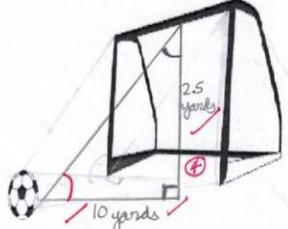
Students' Ability to Solve Mathematics Problems

School A. Inspection of students' responses to the post-intervention assessment items indicated that they were able to produce responses to items that required the recall of definitions and application of algorithms to solve simple word problems, but there was evidence of diversity in their ability to interpret more complex story problems with complete accuracy. However, even given these errors, they were able to accurately select the appropriate trigonometric ratio and perform the computations. The student's response illustrated in

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Figure 10 is completely accurate.

4. Carlos calculates that the perpendicular distance from his football to the bottom of the goal post is 10 yards and the goal post is 2.5 yards high. If he wants to determine the angle he would need to kick the ball to touch the top of goal post



(i) Draw in the right angle triangle formed between the goal post and the ball in the given diagram. Identify the given distances and the right angle. (4marks)

(ii) Calculate the angle he would need to kick the ball to touch the top of the goal post (3 marks)

$\tan \theta = \frac{2.5}{10}$

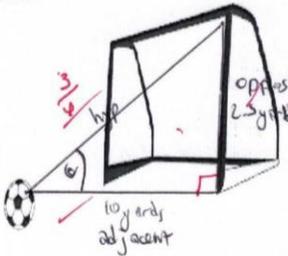
$\theta = \tan^{-1}(0.25)$

$\theta = 14^\circ$

Figure 10 Sample of Accurate Student Response

In comparison, the student's response in Figure 11 is partially accurate. The diagram did not include the right angle and the apex of the triangle not aligning with the top of the goal post. However, the appropriate trigonometric ratio was selected and the computation was accurate.

4. Carlos calculates that the perpendicular distance from his football to the bottom of the goal post is 10 yards and the goal post is 2.5 yards high. If he wants to determine the angle he would need to kick the ball to touch the top of goal post



(i) Draw in the right angle triangle formed between the goal post and the ball in the given diagram. Identify the given distances and the right angle. (4marks)

(ii) Calculate the angle he would need to kick the ball to touch the top of the goal post (3 marks)

$\tan \theta = \frac{2.5}{10}$

$\theta = \tan^{-1}\left(\frac{2.5}{10}\right)$

$\theta = 14.0^\circ$

Figure 11 Sample of Partially Correct Diagram and Accurate Computation

Similarly, the student's response in Figure 12 is partially accurate. The diagram is accurate but the notation used for the trigonometric ratio was inaccurate.

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4. Carlos calculates that the perpendicular distance from his football to the bottom of the goal post is 10 yards and the goal post is 2.5 yards high. If he wants to determine the angle he would need to kick the ball to touch the top of goal post

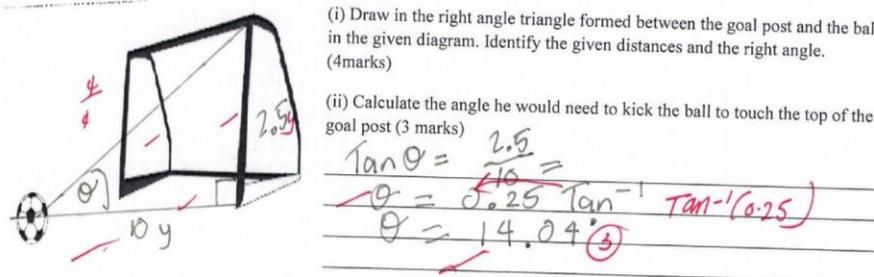


Figure 12 Sample of Correct Diagram and Inaccurate Notation

It was also noted that one student who was not able to accurately produce the diagram from the problem information, was able to select the trigonometric ratio that matched his diagram and begin to formulate a response (see Figure 13).

4. Carlos calculates that the perpendicular distance from his football to the bottom of the goal post is 10 yards and the goal post is 2.5 yards high. If he wants to determine the angle he would need to kick the ball to touch the top of goal post

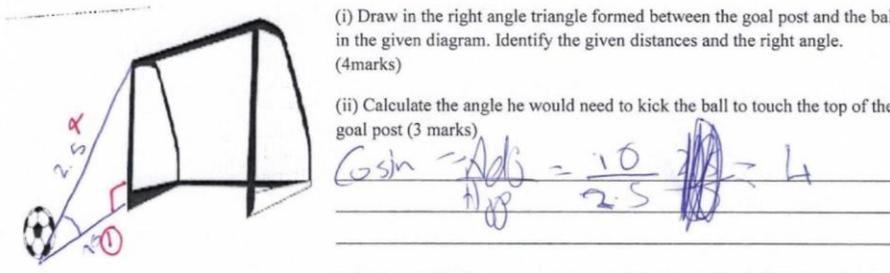


Figure 13 Sample of Inaccurate Student Response

Students' post-test scores indicated an overall increase in means from baseline scores. Figure 14 illustrates an increase in achievement scores post-intervention. A comparison of box plots reveals an increase in the minimum score from 0% to 12%, in the median score from 61.5% to 80%, and an increase in the maximum score from 90% to 100%.

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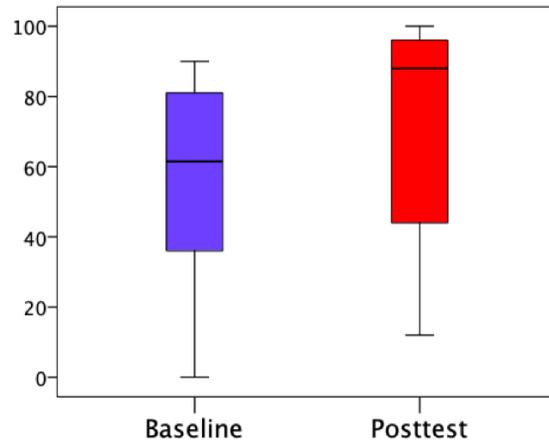


Figure 14 Box Plot Comparison of Students' Baseline and Post-test Means

Additionally, 24 out of 30 students earned higher post-test scores (see Figure 15). It was noted that post-test scores for Students 10, 11 and 24 were below their baseline scores.

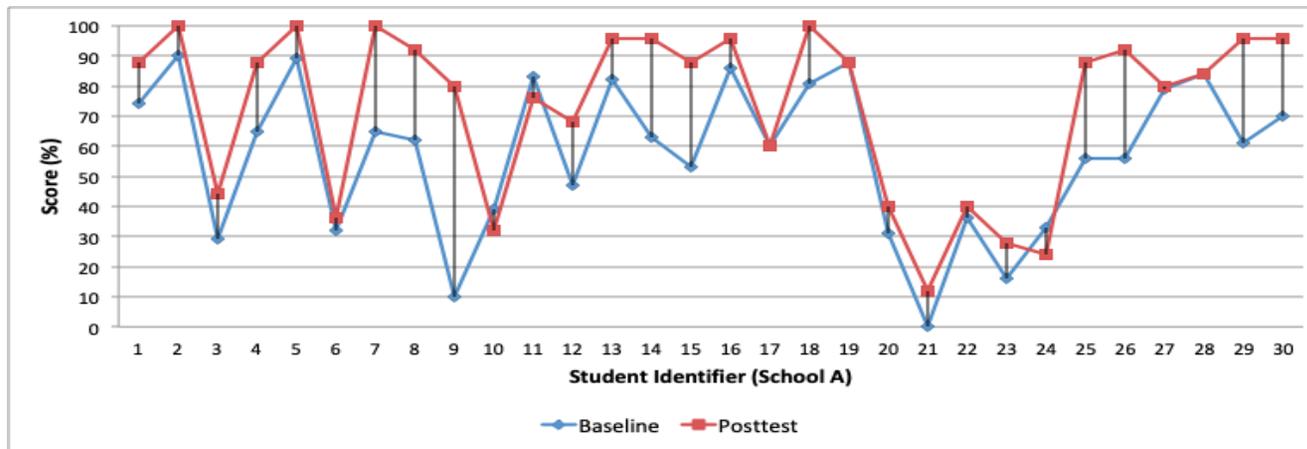


Figure 15 Line Graph Comparison of Students' Baseline and Post-test Means

A paired samples t-test revealed a significant increase, $t(29)=5.213$, $p < 0.05$, from students' baseline means ($M= 57.33$, $SD= 24.74$) to post-test mean scores ($M=73.60$, $SD= 27.10$). Overall, these findings suggest that most students were able to solve problems related to trigonometry, and there was a significant improvement in their mathematics achievement at the end of the intervention.

School B. Inspection of students' responses on the pre-test and post-test also support the increase in their ability to solve problems in trigonometry. For example, although students had been exposed to these concepts in trigonometry in Form 4/Grade 9, they could not solve related problems in the pre-test (Figure 16(a) and Figure 17(a)). However, in the post-test, they were able to solve these problems, though with some computational errors (Figure 16(b) and Figure 17(b)).

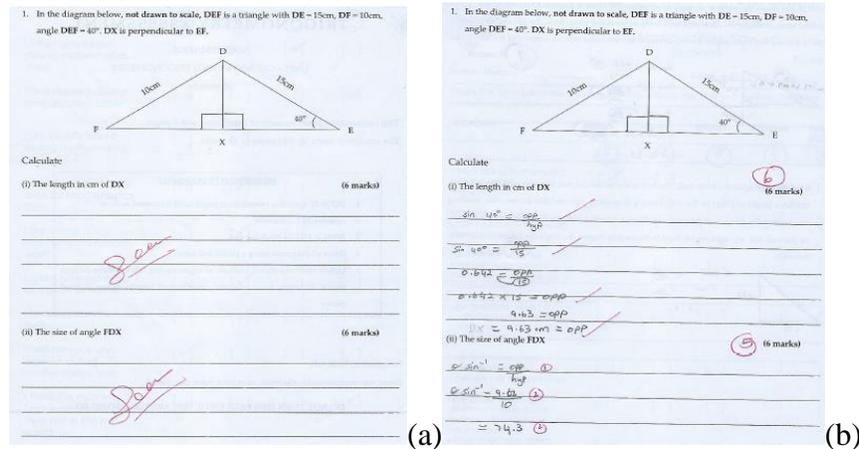


Figure 16 Sample of Student's Response in the Pre-test (a) and Post-test (b)

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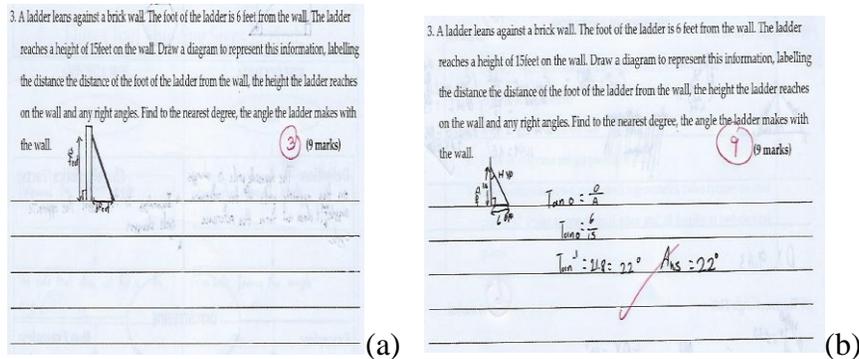


Figure 17 Sample of Student's Response in the Pre-test (a) and Post-test (b)

From the pre-tests and post-tests, with the exception of Student 7, there was an increase in the total scores (see Figure 18). A comparison of box plots reveals no increase in the minimum score because Student 7 scored 0% in both tests, but an increase in the median score from 7% to 37%, and an increase in the maximum score from 77% to 97%.

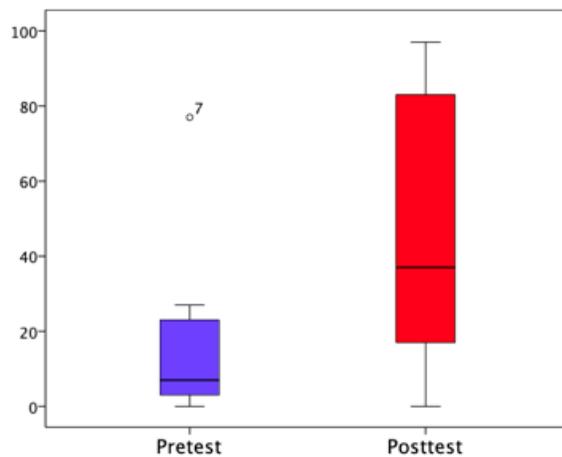


Figure 18 Box Plot Comparison of Students' Pre-test and Post-test Means

From Figure 19, it is evident that all students earned higher post-test scores, except for Student 15, who is particularly weak in

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traditional academic subjects and usually performs better in
vocational subjects.

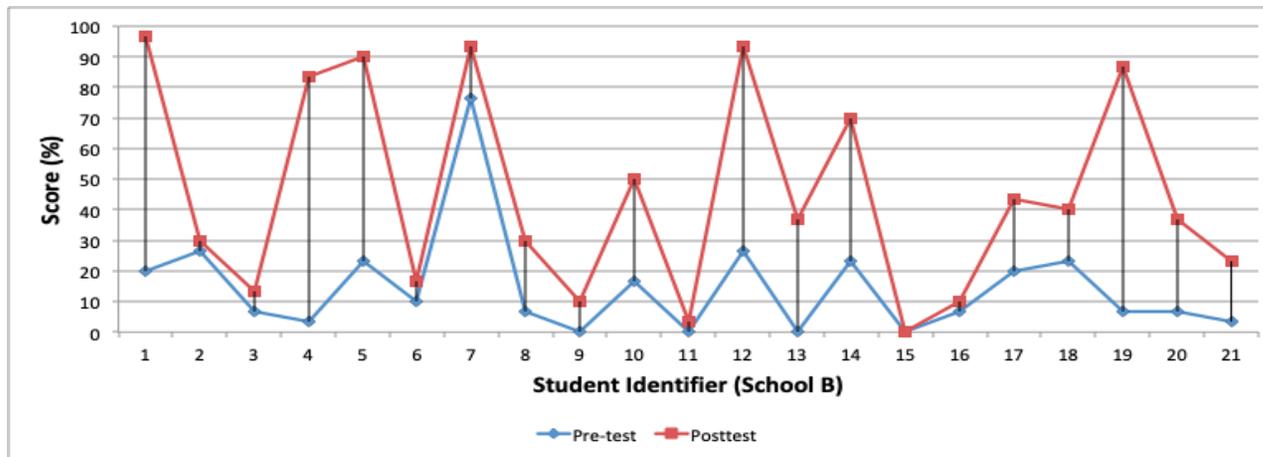


Figure 19. Line Graph Comparison of Students' Baseline and Post-test Means for School B

A paired samples t-test also confirmed an increase in academic performance, $t(21) = 5.137, p < .05$, from pre-intervention ($M=4.38, SD = 5.14$) to post-intervention ($M=13.67, SD = 10.02$). Overall, these findings suggest that all students were better able to solve problems related to trigonometry and there was a significant improvement in their mathematics achievement.

School C. Inspection of students' responses to the post-test items suggested that by the time they wrote the post-test, all of them were able to differentiate between infinite and finite sets, and to list the elements of a given set. Most of them were able to interpret problems in which information was provided about sets in words, and provide responses in words or using set-builder notation. Moreover, they were able to accurately interpret Venn diagrams to identify regions related to the intersection and union of two sets, subsets of sets, the complement of sets, and to list the elements of these sets in words and set-builder notation. Many of them, though not all, were also able to represent information provided in story problems on Venn diagrams. With the exception of five students, students' responses to test items suggested that they could recall what they had learned during instruction. These five students had not attended school regularly and were unable to keep up with the rest of the class.

Students' post-test scores indicated an overall increase in means from baseline scores. Figure 20 illustrates an increase in achievement post-intervention. A comparison of box plots reveals an increase in the minimum score from 22% to 30%, in the median score from 64% to 86% and an increase in the maximum score from 95% to 100%. It is noted that Student 21 scored well below the rest of the class.

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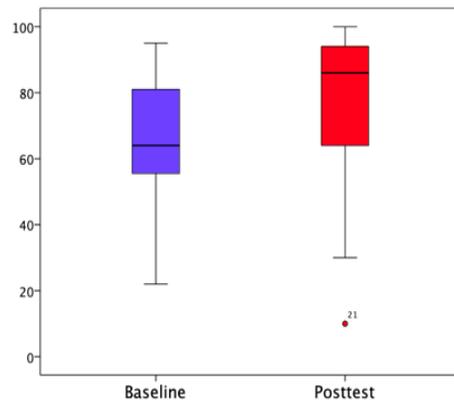


Figure 20 Box Plot Comparison of Students' Baseline and Post-test Means

Pre-intervention, three students' baseline scores were below 50%, but only two students' post-test scores were below 50%. Additionally, four students scored 100% in the post-test and 17 out of 23 students post-test scores were higher than their baseline scores (see Figure 21).

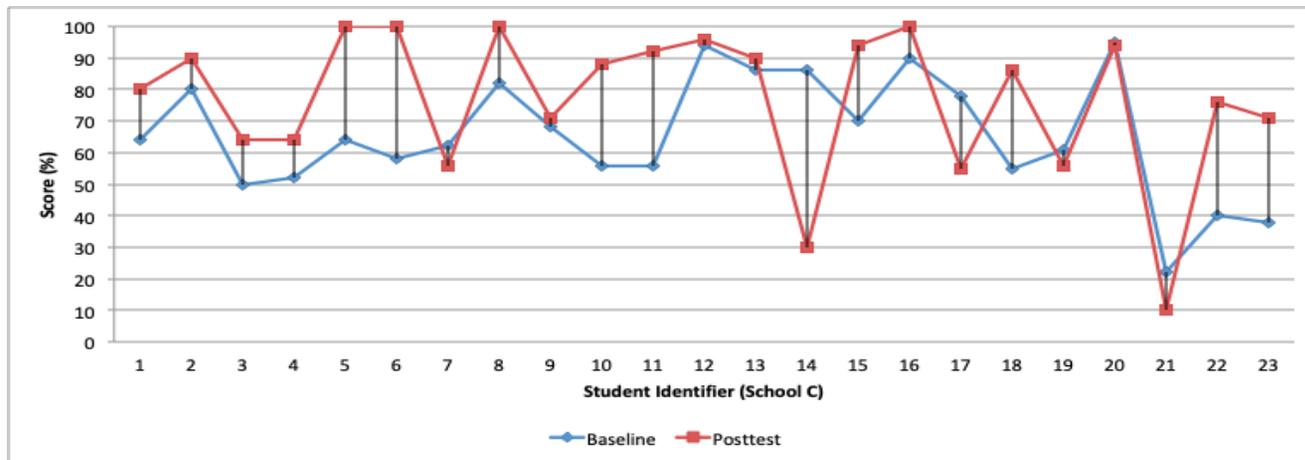


Figure 21 Line Graph Comparison of Students' Baseline and Post-test Means for School C

A paired samples t-test revealed a significant increase, $t(22)=2.264$, $p < 0.05$, from students' baseline means ($M= 65.78$, $SD= 19.10$) to post-test mean scores ($M=76.74$, $SD= 23.75$). These findings suggest that students were better able to solve problems related to set theory and there was a significant improvement in their mathematics achievement.

Synthesis of Findings and Discussion

These three studies explored the outcomes of integrating different types of concrete manipulatives into mathematics instruction in Form 4 (teacher-made geostrips for Trigonometry in School A), Form 5 (commercially available miniature figures, angLegs, dowel rods and foam cubes for Trigonometry in School B), and Form 1 (familiar items like bottle caps, bottles, foam cubes, string, modelling clay and craft sticks for Set Theory in School C) at three secondary schools in Trinidad and Tobago. In the following paragraphs, the authors compare findings across the three school sites to identify similarities and differences and to discuss these with respect to the research literature on concrete manipulatives in mathematics.

It was noted that over time, students' self-reported engagement in learning significantly increased at the end of the intervention. This finding was supported by students' and teachers' journal entries, and teachers' observations of students during instruction. It was concluded that, across the three sites, the use of manipulatives enriched the learning environment by increasing exploration through inductive inquiry, introducing fun and excitement into the learning environment, and strengthening interest and engagement in mathematics learning activities. Teachers noted that students demonstrated more on-task behaviours that allowed them to complete seatwork, and they increasingly interacted with the manipulatives over time. There was a noted decrease in tardiness to class and requests to leave the classroom during lessons over time. Also, students appeared to be better prepared for class with their books and pens, and they more willingly asked and responded to questions over time. Their relationships with other students

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improved as they engaged in mathematical inquiry and cooperation during group activities. They were excited about their tactile learning experiences, once they became accustomed to using the manipulatives for learning.

These changes in observable behaviours are evidence of positive changes across the three dimensions of engagement: affective, behavioural and cognitive (Fredrick et al., 2004). These findings further align with those of Shaw (2002), Ojose and Sexton (2009), Boggan et al. (2010), Siew et al. (2013), Brijlall and Niranjana (2015), Cockett and Kilgour (2015), and Larkin (2016), who suggested that changing the mathematics learning environment by simply introducing concrete manipulatives can demonstrably reduce student boredom, increase the fun and enjoyment they experience during learning and sustain their engagement in learning activities. This is a welcome outcome of these action research studies conducted by teacher-researchers in three secondary schools in Trinidad and Tobago. These teachers had insider knowledge of their contexts and were in the best position to identify interventions that could address issues that influenced teaching and learning there.

The significant increase in students' assessment scores at the end of the intervention suggests that the use of manipulatives helped students connect their concrete explorations with manipulatives, with abstract mathematical concepts and relationships to solve mathematical problems that mimicked daily life. This finding supports Kelly's (2006) suggestion that the strength of concrete manipulatives lies in its ability to help students bridge the gap between the mathematics they learn at school with its applications in solving problems in daily life. In this way, students can develop the ability to represent mathematical ideas in a variety of ways using multiple representations, which are important skills for problem solving, highlighted by Lester and Kehle (2003), Uttal et al. (2009), Karakuş and Peker (2015) and Hakki (2016). This approach creates opportunities for students to develop their conceptual understanding about mathematical topics and relationships before they learn rules and theorems, as suggested by Moyer (2001), Van de Walle et al. (2009), and Laski et al. (2015).

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Across the three schools, students demonstrated the ability to interpret different types of problems - story, short response and multiple choice - and retrieve appropriate concepts and skills from memory to respond correctly, as suggested by Lester and Kehle (2003). These findings concur with Ojose and Sexton (2009) regarding improved achievement after exposure to instruction with manipulatives. This is a critical finding given already established global concerns about declining mathematics achievement and evidence of the same in Caribbean territories noted by CXC (CXC, 2018). It is also noted that though Larkin (2016) suggested that manipulatives are less effective with students older than 12 years, students in these studies across the three schools who ranged in age from 11 years to 17 years, responded favourably to their use. This suggests that there is diversity among learners regarding their stage of development, and some learners well into their teen years may still require the support of visual and tactile sensory experiences when learning complex mathematical concepts and relationships.

Researchers' Reflections

As the research team reflected on the outcomes of these three studies, they noted that although these studies were localised to three secondary schools, these schools are similar to other schools across Trinidad and Tobago. It is likely, therefore, that similar findings could be realised in other schools across Trinidad and Tobago under similar research conditions, barring specific unique characteristics of schools that make them substantially different from the three involved in these studies. Therefore, the research team suggests that other teacher-researchers in schools across Trinidad and Tobago conduct their own research into the integration of concrete manipulatives into mathematics instructions to compare their findings with those of these studies, regarding their influence on student engagement in learning and mathematics problem solving as presented in this paper. The procedures and methods articulated in this paper provide sufficient detail for other educators and researchers to replicate, adapt or enhance them to align with

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students' needs at other schools and the nuances among different schools in Trinidad and Tobago. Further, such investigations are warranted across grade levels as well, particularly because the findings presented in this paper suggest that even older learners, who are assumed to have moved beyond the concrete stage of development, could respond favourably to concrete manipulatives in learning mathematics.

The findings presented in this paper provide evidence of improvement in students' ability to solve mathematics problems. This was evident in students' ability to express their thinking through diagrams in the absence of the concrete manipulatives in the assessment, a skill that the teachers helped students develop by gradually removing the scaffold of the concrete manipulative over time. Students developed the ability to interpret simple real-life problems and model them diagrammatically, bypassing the need to use concrete representations through manipulatives. This brings to the fore, the potential of having students work with concrete manipulatives to model and investigate mathematical relationships in preparation for transitioning to diagrammatic representations of these relationships. It also highlights the importance of concrete manipulatives in assisting students to utilise multiple representations in mathematics problem solving.

Reflections on the interventions at these schools highlight how resourceful teachers can be when planning instruction to meet the needs of their students. Given the financial constraints that were evident at the schools in these studies, and the limited availability of commercially-produced manipulatives, the teachers purchased material and used readily available items to make manipulatives for their students to use. Unfortunately, this is the reality for many teachers who are constrained by school resources, and resort to personally purchasing or otherwise accessing teaching aids. Admittedly, it was difficult to provide each student with her/his own set of manipulatives, so students worked together with the manipulatives in small groups. This introduced the social aspect of learning which allowed students to discuss their mathematical ideas, express mathematics problems using various representations, think divergently, work collaboratively, and gain confidence in their

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mathematics problem solving skills. This is a testament to the creativity of these teacher-researchers, and the concern they have for their students, whom they recognised could benefit from tactile learning experiences. As it was for the teachers involved in these studies, for many teachers, the internet can be a good source for ideas about teaching resources, which they can create themselves or with the involvement of their students.

The research team noted the absence of mathematics rooms or spaces dedicated to the teaching and learning of mathematics in all of the three schools in this study. Consequently, the teacher-researchers spent considerable time at the start of the lessons arranging the physical space of their classrooms to facilitate group work with manipulatives. They then returned the rooms to their original condition after each lesson. They understood the importance of preparing the classrooms to create learning environments that facilitated the use of concrete manipulatives, and their effort highlights the value they placed on creating learning spaces dedicated to inductive inquiry in mathematics. Lack of a specialised mathematics room is a reality in many schools across Trinidad and Tobago, and even in some schools where these dedicated spaces are available, they may be under-resourced and under-utilised. In light of these realities teachers must be creative in the use of available spaces for the teaching of mathematics, to stimulate students' interest in learning mathematics, and engage them in meaningful learning experiences that develop their mathematics problem solving.

As a final reflection, the research team acknowledges that further research into the teaching and learning of mathematics should be a research imperative due to the already established concern about the decline in students' mathematics achievement internationally (see, Stokke, 2015; Wolfram, (2014), and regionally (see, Budoo, 2017; CXC, 2018; Kalloo & Mohan, 2015). The team notes that although these studies revealed favourable students' responses to concrete manipulatives in terms of engagement in learning and problem solving, further research is needed to

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substantiate the claims made in this paper at other schools in Trinidad and Tobago. Further, the team did not explore the connection between increasing students' engagement in learning and improving mathematical problem solving, and strongly advocates for such research on a much larger scale across Trinidad and Tobago, to determine whether such a connection can be made.

Conclusion

The outcomes of these studies suggest that students responded favourably to the use of manipulatives during the intervention, regardless of their age, grade level, and school type. Students were more engaged in learning mathematics during the intervention and improved their mathematics problem solving techniques. The use of concrete objects provided visual and kinaesthetic sensory experiences for students. It enhanced their engagement in learning and their understanding of mathematical concepts through inquiry and discovery of properties and relationships among them. This helped them to solve problems in the post-intervention assessments. This intervention provides teachers with a learner-centred instructional approach that actively engages students' imagination and creativity. Therefore, like Boggan et al. (2010) and Kontaş (2016), this research team supports the integration of concrete manipulatives into mathematics instruction in secondary schools in Trinidad and Tobago.

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